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Time-varying investment dynamics in the USA

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Ivan Mendieta-Muñoz*

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We study the time-varying effects of Tobin's q and cash flow on investment dynamics in the USA using a vector autoregression model with drifting parameters and stochastic volatilities estimated via Bayesian methods. We find significant variation over time of the response of investment to shocks in both variables. The time-varying sensitivity of investment to a shock in Tobin's q (cash flow) decreased (increased) since the early 1960s through the early 1980s, increased (decreased) since the early 1980s through the early 2000s, and it has decreased (increased) importantly again since then. Our results show that, although Tobin's q and cash flow are complementary sources of information for investment decisions, their relative importance for investment dynamics has varied considerably over time, so both variables also represent alternative sources of information for short-run fluctuations in investment.

Keywords: Investment dynamics, Tobin's q , cash flow, time-varying parameters, vector autoregression, stochastic volatility.

JEL Classification: C11, C32, E22, E32, G31.

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1. Introduction

The theoretical and empirical literature on aggregate investment has long emphasized that its dynamics are heavily influenced by Tobin’s q —a variable related to the availability of external equity finance—and cash flow—a variable associated with the availability of internal funds. Importantly, recent empirical literature has discussed that the investment sensitivities to both variables can vary dynamically and non-linearly over time.

This paper studies the evolution of the sensitivity of aggregate investment to Tobin’s q and cash flow in the USA during the post-World War II period. To do so, we consider a multiple equation model that introduces time variation in the dynamic structural linkages among the variables and that captures the possible heteroskedasticity of the shocks in the simultaneous relations of interest. Hence, we estimate a time-varying parameter vector autoregression model with stochastic volatility (TVP-VAR-SV model) via Bayesian methods along the lines of [Primiceri \(2005\)](#) and [Del Negro and Primiceri \(2015\)](#). This empirical model allows for a flexible strategy to study the possible time-varying behavior of the underlying structure of investment dynamics in a multivariate framework.

Our main findings can be summarized as follows. First, the results show significant variation over time of the response of investment to shocks in Tobin’s q and cash flow. Second, the time-varying response of investment to a shock to Tobin’s q is almost the mirror image to the time-varying response of investment to a shock to cash flow. Specifically, the time-varying sensitivity of investment to a shock in Tobin’s q decreased since the early 1960s through the early 1980s, increased since the early 1980s through the early 2000s, and it has tended to decrease importantly again since then. On the other hand, the time-varying sensitivity of investment to a shock in cash flow increased since the early 1960s through the early 1980s, decreased since the early 1980s through the early 2000s, and it has tended to increase importantly again since then. In this sense, the main findings suggest that, although Tobin’s q and cash flow represent complementary sources of information for investment decisions, their relative importance for investment dynamics has changed considerably over time, so Tobin’s q and cash flow are also alternative sources of information needed to understand short-run investment fluctuations.

The remainder of the paper is organized as follows. A brief review of the literature that helps to motivate the current research is presented in [section 2](#). [Section 3](#) presents the relevant data and discusses some stylized facts that additionally motivate the use of a TVP-VAR-SV model

to study the dynamics of aggregate investment. The model and the empirical methodology are summarized in section 4. Section 5 presents the main results; while section 6 conducts a series of robustness checks. Finally, the main conclusions are presented in section 7.

2. Related literature

The present contribution is mainly related to the recent theoretical and empirical literature that has emphasized that investment depends dynamically and non-linearly on variables associated with liquidity and finance constraints. At the theoretical level, the models developed by [Lettau and Ludvigson \(2002\)](#), [Abel and Eberly \(2011\)](#) and [Abel and Eberly \(2012\)](#) articulate different possibilities to understand the relationships between investment, Tobin's q and cash flow. This literature has shown three important results. First, there are significant dynamic interactions that can change over time between investment and Tobin's q —for instance, because discount rates are not constant ([Lettau and Ludvigson, 2002](#)). Second, even if other adjustment costs and financial constraints are eliminated, it can be shown that investment still remains sensitive to both Tobin's q and cash flow ([Abel and Eberly, 2011](#)). Third, when growth options that vary over time are considered—which occurs because the firm's level of productivity is a choice variable, investment is positively correlated with cash flow during intervals of time between consecutive technology upgrades, but investment would be uncorrelated with Tobin's q during such intervals; whereas the positive correlation between investment and Tobin's q is essentially associated with the forward-looking nature of the value of the firm, which can also change over time ([Abel and Eberly, 2012](#)).

At the empirical level, recent contributions have explored the possible changes over time of the sensitivity of investment to Tobin's q and cash flow, mainly at the micro level by considering firm-level data. [Ağca and Mozumdar \(2008\)](#), [Brown and Petersen \(2009\)](#) and [Chen and Chen \(2012\)](#) use US manufacturing firm data for the periods 1970-2001, 1970-2006 and 1967-2006, respectively. [Ağca and Mozumdar \(2008\)](#) controls for other factors associated with capital market imperfections—namely, fund flows, institutional ownership, analyst following, bond ratings, and an index of antitakeover amendments, finding a steady decline in the estimated investment-cash flow sensitivity and a relatively stable investment-Tobin's q sensitivity. [Brown and Petersen \(2009\)](#) is interested in studying how R&D investment and developments in equity markets have impacted the investment-cash flow sensitivity, finding an

important decline of the latter over time and also a smaller decline in the investment-Tobin's q sensitivity. [Chen and Chen \(2012\)](#) find that the investment-cash flow sensitivity declined over their entire sample period (and even completely disappeared during the 2007-9 Great Financial Crisis); while the investment-Tobin's q sensitivity has remained relatively stable.

In the same vein, [Mclean and Zhao \(2014\)](#) conduct their analysis using a sample of US firms for the period 1965-2010, showing that investment is more sensitive to Tobin's q (cash flow) during expansions (recessions). Both [Lewellen and Lewellen \(2016\)](#) and [Grullon et al. \(2018\)](#) emphasize the relevance of cash flow for investment decisions using a sample of US nonfinancial firms for the periods 1971-2009 and 1950-2011, respectively; however, while the former suggests that this sensitivity has decreased, the latter finds that the investment-cash flow sensitivity has increased for the largest 100 investing firms—which are the ones that explain approximately 60% of the total variation in aggregate investment.

Using quarterly aggregate data for the US economy, [Gallegati and Ramsey \(2013\)](#) and [Verona \(2020\)](#) employ wavelet analyses to study investment dynamics. Without considering cash flow, [Gallegati and Ramsey \(2013\)](#) find important evidence of instability regarding the investment-Tobin's q relationship for the period 1952-2009, which even becomes negative during the 1980s. [Verona \(2020\)](#) considers the influence of both cash flow and Tobin's q , finding that the investment-Tobin's q sensitivity has declined during the period 1952-2017, while the investment-cash flow sensitivity has declined at business cycle frequencies but it has tended to remain stable at lower frequencies (medium-to-long run).

Our article contributes to the aforementioned literature as follows. First, we focus on the analysis of investment dynamics at the aggregate level by using a vector autoregression (VAR) model—that is, a multiple equation modeling approach. Although micro level studies and the use of firm-level data are important to capture the potential heterogeneity of investment decisions, for example, it is challenging to capture the relevant dynamic interactions as well as the structural feedback effects between the variables of interest using these methodologies—which helps to explain some of the considerably different results reported by this strand of literature.

Second, the incorporation of time-varying parameters (TVPs) into the VAR model represents a highly flexible framework for the estimation and interpretation of time variation in the systematic and non-systematic components of investment and its relationship to Tobin's q and cash flow compared to rolling regressions—which are widely employed by micro level studies, but that are known to lead to unreliable results in terms of spurious non-linear coefficient patterns.

Third, incorporating stochastic volatilities besides TVPs into the VAR model allows us to control for the possible heteroskedasticity of the shocks that have taken place during the post-World War II period (*i.e.*, the Great Moderation)—which allows to provide a comprehensive characterization of the possible uncertainty around the estimates.

In this sense, the TVP-VAR-SV model proposed to study aggregate investment dynamics complements directly the analysis of [Verona \(2020\)](#). Although our modeling approach only captures the short-run dynamic interactions between investment, Tobin’s q and cash flow, we are able to go beyond the time-varying correlations derived from his analysis, thus providing a time-varying structural interpretation of the short-run dynamic interactions between the variables of interest.¹

3. Data and stylized facts

We use the same variables used by [Verona \(2020\)](#), thus focusing on the interactions among three variables: the investment rate (i_t), Tobin’s q (q_t) and cash flow (c_t). We use quarterly time series data for the USA over the period 1951:Q1-2022:Q4, selected according to the availability of data. [Figure 1](#) shows the time series plots of i_t , q_t and c_t with National Bureau of Economic Research (NBER)-dated recession dates in shaded bars. The series were constructed as follows. The i_t series corresponds to aggregate private non-residential fixed investment as a percentage of aggregate capital.² The q_t series corresponds to Tobin’s q of the nonfinancial corporate business sector, constructed as corporate equities as a percentage of net worth.³ The c_t series is corporate profits as a percentage of GDP.⁴

From [figure 1](#) it is possible to observe that the procyclical i_t has experienced substantial fluctuations over time. There have been two notable investment booms: the one starting in the early 1960s, which ended before the recession of 1969-70, and the one starting in the early 1990s,

¹Indirectly, our paper also complements the recent contributions by [Haque et al. \(2021\)](#) and [Mendieta-Muñoz and Sündal \(2022\)](#). [Haque et al. \(2021\)](#) also use a TVP-VAR-SV model, but their interest consists in studying the effects of financial uncertainty shocks on investment, so they do not consider either the effects of Tobin’s q or cash flow in their empirical analysis. On the other hand, [Mendieta-Muñoz and Sündal \(2022\)](#) also study the possible nonlinear dynamic effects of investment; but they consider: (i) a threshold VAR modeling approach instead of TVPs to capture nonlinearities; and (ii) the effects of credit spreads instead of Tobin’s q .

²This time series was extracted from [Amit Goyal’s website](#) (last accessed on September 6th, 2023), which follows the methodology of [Welch and Goyal \(2008\)](#).

³We used the Federal Reserve Economic Data (FRED) data base of the Federal Reserve Bank of St. Louis: corporate equities correspond to the ‘NCBELLQ027S’ series and net worth corresponds to the ‘TNWMVBSNNCB’ series.

⁴Data was obtained from the National Income and Product Accounts (NIPA) of the Bureau of Economic Analysis (BEA): nominal corporate profits with inventory valuation adjustment and capital consumption adjustment were extracted from Table 1.12 (line 13); while nominal GDP was extracted from Table 1.1.5 (line 8).

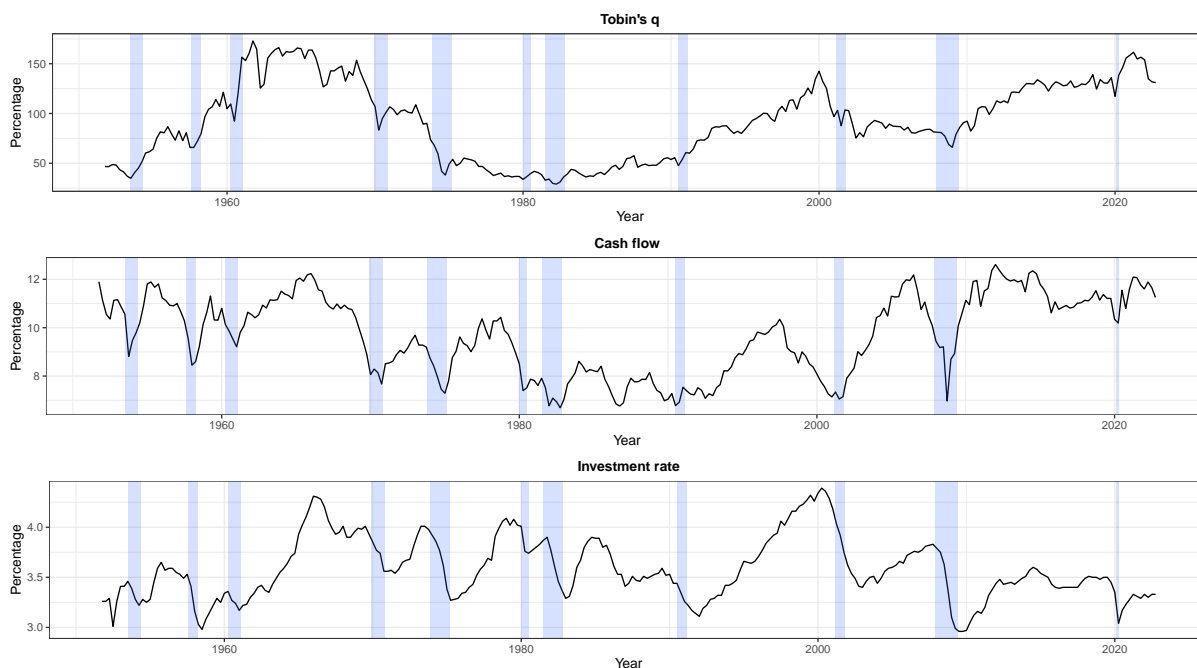


Figure 1: USA, 1951:Q1-2022:Q4. Time series plots of Tobin's q , cash flow, and investment rate. Shaded areas indicate NBER recessions dates.

which ended just before the 2001 recession associated with the dot-com bubble. As discussed by other contributions, investment has been declining since then, especially since the Great Recession of 2007-9 (Gutiérrez and Philippon, 2017).

During this period, the behavior of q_t and c_t has also changed considerably over time, which suggests that their respective effects on i_t have been time-varying. Both variables exhibited relatively high levels before the 1970s, which coincides with the first investment boom, and declined up until the recession of 1990-1 (although q_t rose since the mid-1980s), which also coincides with the trajectory of i_t . However, q_t is the only variable that experienced a clear sustained increase during the early 1990s, thus suggesting that the second boom in i_t was mainly driven by this effect. Since the 2001 recession, both q_t and c_t have tended to show high levels—the only exceptions being during the global financial crisis of 2007-9 and the COVID-19 recession of 2019-20; whereas i_t has experienced lower levels.

To illustrate these points further, figures 2 and 3 show the scatter plots between i_t and q_t and i_t and c_t , respectively, for six different sub-periods—which, broadly speaking, try to capture the effects across different decades. There is considerable heterogeneity regarding the interactions between the variables. For example, although the positive correlation between i_t and q_t is almost always corroborated, the association between these two variables is nonexistent during 1970:Q1-1979:Q4, this association is negative during 1980:Q1-1989:Q4, and the correlation also

seems to be weak in the most recent period (2010:Q1-2022:Q4). Likewise, i_t and c_t seem to be positively correlated during most sub-periods; however, the association is negative during 2000:Q1-2009:Q4. There is also considerable variation regarding the constructed confidence intervals across the sub-periods shown in figures 2 and 3, which suggests important time variation with regards to the precision of the estimated effects.

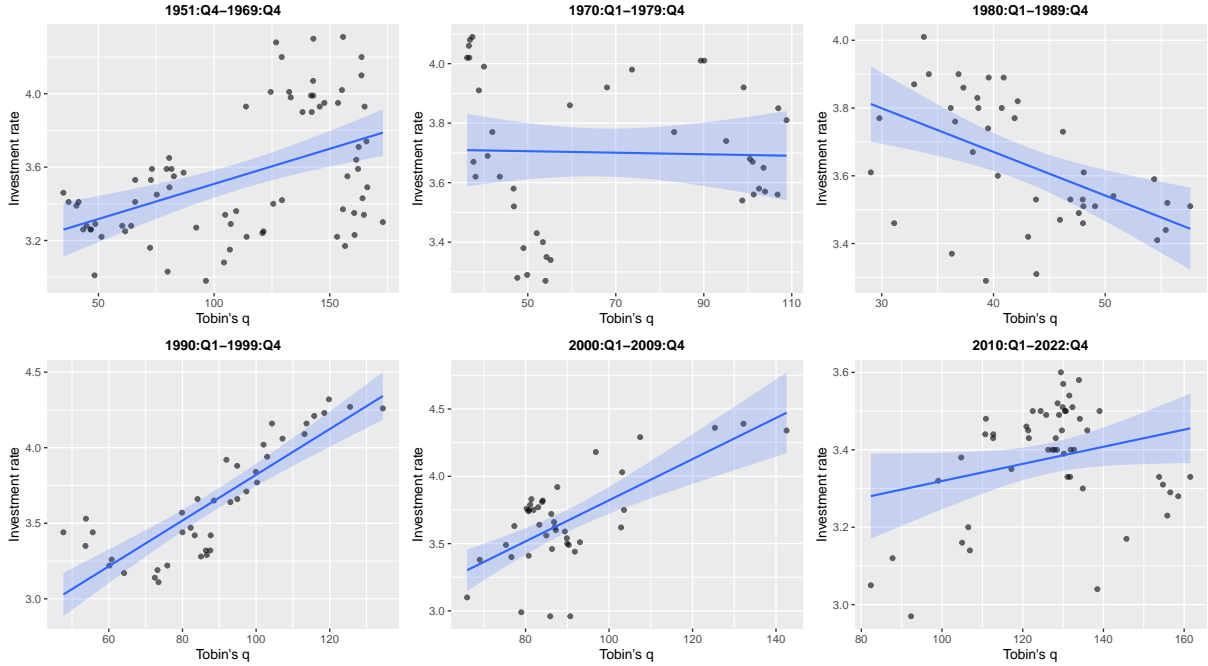


Figure 2: USA, 1951:Q1-2022:Q4. Scatter plots between investment rate and Tobin's q for different sub-periods. Straight lines show OLS regression lines. Shaded areas indicate 95% confidence level intervals of the regression lines.

The response of i_t to both q_t and c_t considering the different sub-periods is summarized by the regression analyses shown in table 1. First, the results show that, overall, i_t is more sensitive to c_t than to q_t . Second, the sensitivity of i_t to both q_t and c_t has been different across the different sub-periods. For instance, the effect of c_t on i_t is statistically non-significant from 1990:Q1 through 2009:Q4, which corresponds to the two sub-periods where the largest statistically significant effects of q_t on i_t can be found.

The stylized facts presented in this section suggest that the dynamics of i_t during the post-World War II period have been influenced by time-varying effects associated with both q_t and c_t . In other words, the sensitivity of i_t to these two variables seems to be time-varying, so that the relevance of q_t and c_t for investment decisions has been changing over time. Motivated by this evidence, we use a TVP-VAR-SV model to formally study the interactions between the three variables and, most importantly, to capture the possible structural time-varying effects of both

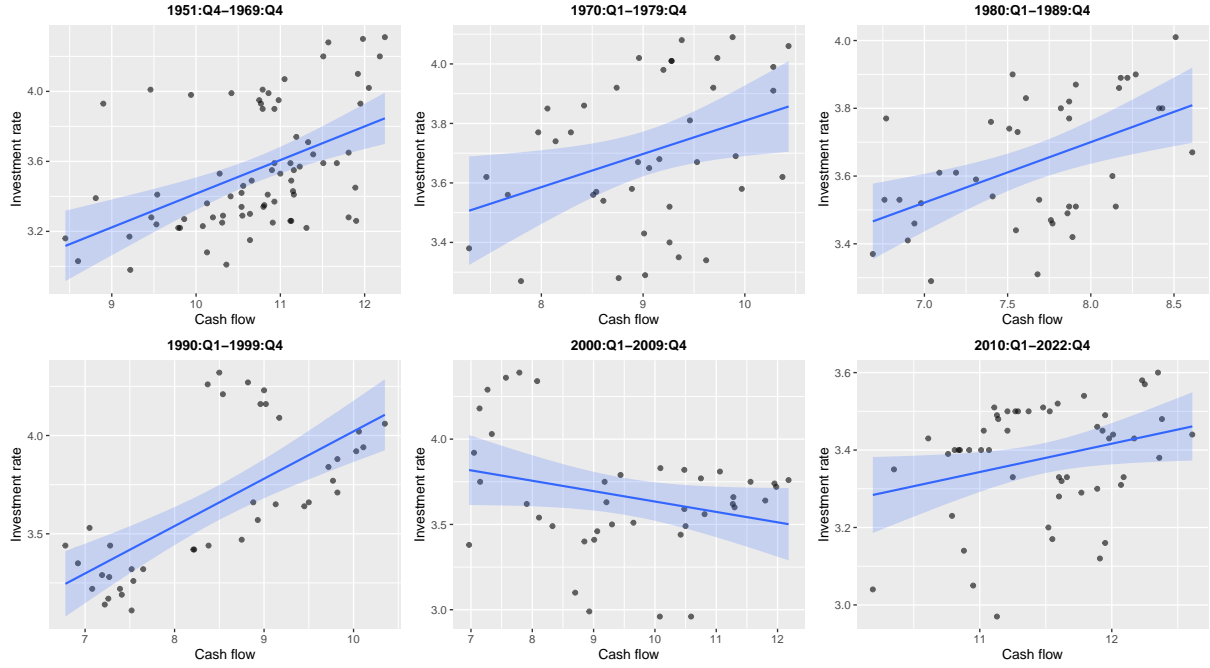


Figure 3: USA, 1951:Q1-2022:Q4. Scatter plots between investment rate and cash flow for different sub-periods. Straight lines show OLS regression lines. Shaded areas indicate 95% confidence level intervals of the regression lines.

Table 1: INVESTMENT RATE EQUATION

Period	Intercept	Tobin's q	Cash flow
1951Q4-1969Q4	1.548** (0.587)	0.003** (0.001)	0.155** (0.057)
1970Q1-1979Q4	2.551** (0.497)	0.001 (0.002)	0.121* (0.051)
1980Q1-1989Q4	2.903** (0.493)	-0.012** (0.003)	0.161** (0.054)
1990Q1-1999Q4	1.872** (0.361)	0.012** (0.003)	0.079 (0.041)
2000Q1-2009Q4	2.359** (0.389)	0.015** (0.002)	-0.005 (0.030)
2010Q1-2022Q4	2.168** (0.563)	0.002 (0.002)	0.079* (0.037)

Notes: We report the OLS regression coefficients of investment as a function of Tobin's q and cash flow for different sub-periods. Heteroscedasticity and autocorrelation consistent (HAC) standard errors are shown in parenthesis. * and ** denote significance at the 5% and 1% level, respectively.

q_t and c_t on i_t in a dynamic modeling framework that also controls for the possible time-varying volatility of the shocks.

4. The empirical model

A reduced form TVP-VAR-SV model of order p can be expressed as:

$$\mathbf{y}_t = \mathbf{C}_t + \sum_{p=1}^P \mathbf{B}_{p,t} \mathbf{y}_{t-p} + \mathbf{u}_t, \quad \mathbf{u}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{\Omega}_t), \quad t = 1, \dots, T, \quad (1)$$

where \mathbf{y}_t is an $n \times 1$ vector of endogenous variables; \mathbf{C}_t is an $n \times 1$ vector of intercepts; $\mathbf{B}_{p,t}$ is an $n \times n$ matrix that contains the p^{th} lag autoregressive coefficients; and \mathbf{u}_t are the heteroskedastic reduced form shocks with time-varying variance-covariance matrix $\mathbf{\Omega}_t$.

We can rewrite equation (1) as follows:

$$\mathbf{y}_t = \mathbf{X}_t \beta_t + \mathbf{A}_t^{-1} \mathbf{\Sigma}_t \varepsilon_t, \quad t = P + 1, \dots, T, \quad (2)$$

where $\mathbf{X}_t \equiv \mathbf{I}_n \otimes (1, \mathbf{y}'_{t-1}, \dots, \mathbf{y}'_{t-P})$, such that \otimes denotes the Kronecker product; the vector β_t is formed by stacking the elements of \mathbf{C}_t and $\mathbf{B}_{p,t}$ equation by equation, so that $\beta_t \equiv \text{vec}([\mathbf{C}_t, \mathbf{B}_{1,t}, \dots, \mathbf{B}_{P,t}]')$; \mathbf{A}_t^{-1} is a lower-triangular matrix with ones on the main diagonal and time-varying off-diagonal elements; $\mathbf{\Sigma}_t$ is a time-varying diagonal matrix that contains the standard deviations of the structural shocks; and $\varepsilon_t \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_n)$ is the vector of standardized structural shocks, such that \mathbf{I}_n is an n -dimensional identity matrix.

Hence, as in [Primiceri \(2005\)](#), the model depicted by equation (2) incorporates two types of parameter instability: time-varying parameters via β_t (which captures the reduced form coefficients) and \mathbf{A}_t^{-1} (which captures the simultaneous relationships between the endogenous variables), as well as time-varying covariance terms via $\mathbf{\Sigma}_t$ (which captures the stochastic volatility of structural shocks).

The dynamics of the model's time-varying parameters are specified as follows:

$$\beta_t = \beta_{t-1} + \nu_t, \quad (3)$$

$$\alpha_t = \alpha_{t-1} + \zeta_t, \quad (4)$$

$$\log \sigma_t = \log \sigma_{t-1} + \eta_t, \quad (5)$$

where $\alpha_t = (a_{21,t}, \dots, a_{nn-1,t})'$ is the vector of non-zero and non-unitary elements of \mathbf{A}_t (*i.e.*, the lower-triangular elements of \mathbf{A}_t) stacked by rows; $\sigma_t = (\sigma_{1,t}, \dots, \sigma_{n,t})'$ is the vector of the main diagonal elements of $\mathbf{\Sigma}_t \mathbf{\Sigma}_t'$; and $\{\nu_t, \zeta_t, \eta_t\}$ are i.i.d. Gaussian random shocks. Thus, equations (3)

through (5) show that we assume that the parameters follow random walk processes—a flexible modeling assumption that allows us to capture both gradual and sudden structural changes.

Let us now define $\psi = (\varepsilon_t, \nu_t, \zeta_t, \eta_t)'$. Following Primiceri (2005), we assume that $\psi \sim \mathcal{N}[\mathbf{0}, \text{diag}(\mathbf{I}_n, \mathbf{Q}, \mathbf{S}, \mathbf{W})]$, so ψ is jointly normally distributed with mutually uncorrelated white noise shocks, zero mean, and variances defined by \mathbf{I}_n and the hyper-parameters \mathbf{Q} , \mathbf{S} and \mathbf{W} , such that:

$$\mathbf{V} = \text{Var} \left(\begin{bmatrix} \varepsilon_t \\ \nu_t \\ \zeta_t \\ \eta_t \end{bmatrix} \right) = \begin{bmatrix} \mathbf{I}_n & 0 & 0 & 0 \\ 0 & \mathbf{Q} & 0 & 0 \\ 0 & 0 & \mathbf{S} & 0 \\ 0 & 0 & 0 & \mathbf{W} \end{bmatrix}, \quad (6)$$

where \mathbf{Q} , \mathbf{S} , and \mathbf{W} are all diagonal positive semi-definite matrices that represent the variance-covariance matrices of shocks to β_t , \mathbf{A}_t , and $\log \sigma_t$, respectively.

We rely on Markov chain Monte Carlo (MCMC) methods to estimate the TVP-VAR-SV model outlined above. First, we follow Primiceri (2005)'s sampling algorithm, but we modify the latter by incorporating the correction noted by Del Negro and Primiceri (2015). Second, we use the same prior distributions and initial states of the parameters distributions employed by Primiceri (2005).⁵ Third, using the relevant MCMC algorithm, we collect 205,000 posterior samples and discard the first 5,000 draws to ensure the convergence of the chain. Fourth, as in Primiceri (2005), we use $p = 2$, so that we estimate the TVP-VAR-SV model considering two lags.⁶

5. Results

Our baseline results reported in this section consider the following ordering of variables: $\mathbf{y}_t = (q_t, c_t, i_t)'$. This implies that we order q_t in \mathbf{y}_t first, c_t second, and i_t last.⁷ Hence, we assume that a shock in q_t effects c_t and i_t contemporaneously; a shock to c_t effects only i_t within the same period; and i_t does not effect c_t and i_t contemporaneously—it only does so with a lag. In short, this ordering of variables reflects that we believe that the availability of external equity finance approximated by q_t is the most exogenous variable in the system, followed by the availability of internal funds approximated by c_t ; while i_t is the most endogenous variable in the system.

⁵Since the use of these priors and the implementation of the relevant sampling algorithm is standard, we summarize the relevant technical details in appendices A and B, respectively.

⁶The baseline results reported in the following section remained unchanged when we considered $p = 3$ instead.

⁷In other words, we employ a recursive identification scheme to identify the relevant shocks based on the Cholesky factorization of the reduced form residual's variance-covariance matrix and the lower-triangular identification scheme imposed via the \mathbf{A}_t^{-1} matrix.

Figure 4 shows the posterior mean together with the 16th and 84th percentiles of the time-varying standard deviation of the structural shocks—that is, the stochastic volatility components aimed at capturing the heteroscedasticity of the shocks as a possible source of time variation. The results indicate substantial time variation in the volatility of shocks, which means that some of the variation in the dynamics of the model is associated with the time variation of the variance-covariance matrix besides the coefficients in the model. Specifically, it is possible to observe that: (i) the volatility of the shocks from the c_t equation is the largest one; (ii) the volatilities of the shocks from the q_t and c_t equations are considerably more persistent than the volatility of the shocks from the i_t equation; (iii) the volatility of the shocks from the q_t equation has decreased over time; and (iv) the volatility of the shocks from the c_t equation has increased over time.

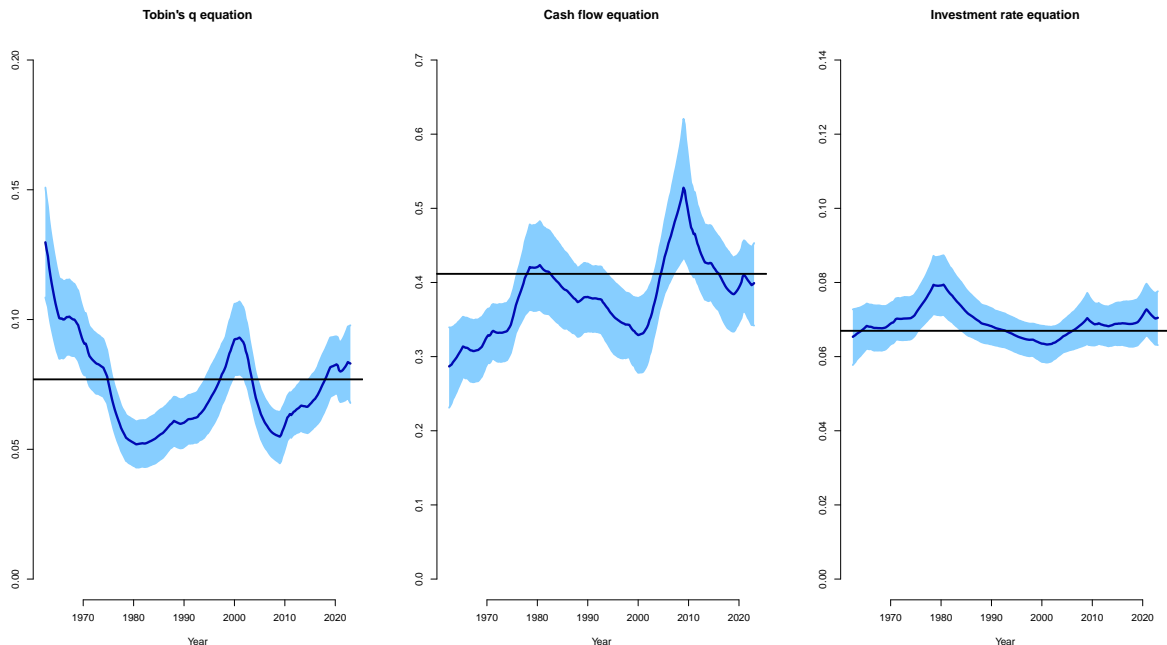


Figure 4: Posterior means of the standard deviations of residuals obtained from the TVP-VAR-SV model. We report the time series plots of the means of the standard deviations of the residuals of Tobin’s q equation, cash flow equation, and investment rate equation in the TVP-VAR-SV model. Shaded areas show the 16th and 84th percentiles. Black horizontal lines show to the means of the standard deviations of the residuals obtained from a standard VAR model (without TVP or SV) estimated via frequentist methods.

Since our main interest consists in studying the possible time-varying effects of q_t and c_t on i_t , we only focus on the responses of the latter to shocks in q_t and c_t . To get an idea of the changing nature of the effects of q_t on i_t that allows for estimation uncertainty by considering error bands, figure 5 shows the impulse responses following a shock to q_t originating in three different dates:

1969:Q4, 2000:Q3 and 2014:Q3. The first two dates are chosen as these correspond to the two quarters where i_t was the highest in the sample; while 2014:Q3 is chosen as a random quarter after the Great Recession of 2007-9 but before the COVID-19 recession of 2019-20. As expected, a positive shock in q_t increases i_t in the three selected dates; however, the response of i_t to a shock in q_t is both larger and more persistent in 2000:Q3. To further illustrate these effects, figure 6 plots the differences between the impulse responses in 1969:Q4 and 2000:Q3, 1969:Q4 and 2014:Q3, and 2000:Q3 and 2014:Q3. Since the error bands do not enclose the zero line for the difference between the impulse response in 2000:Q3 and 2014:Q3, then we can conclude that the impulse response is significantly stronger in 2000:Q3 than in 2014:Q3 (see the bottom-left panel in figure 6).

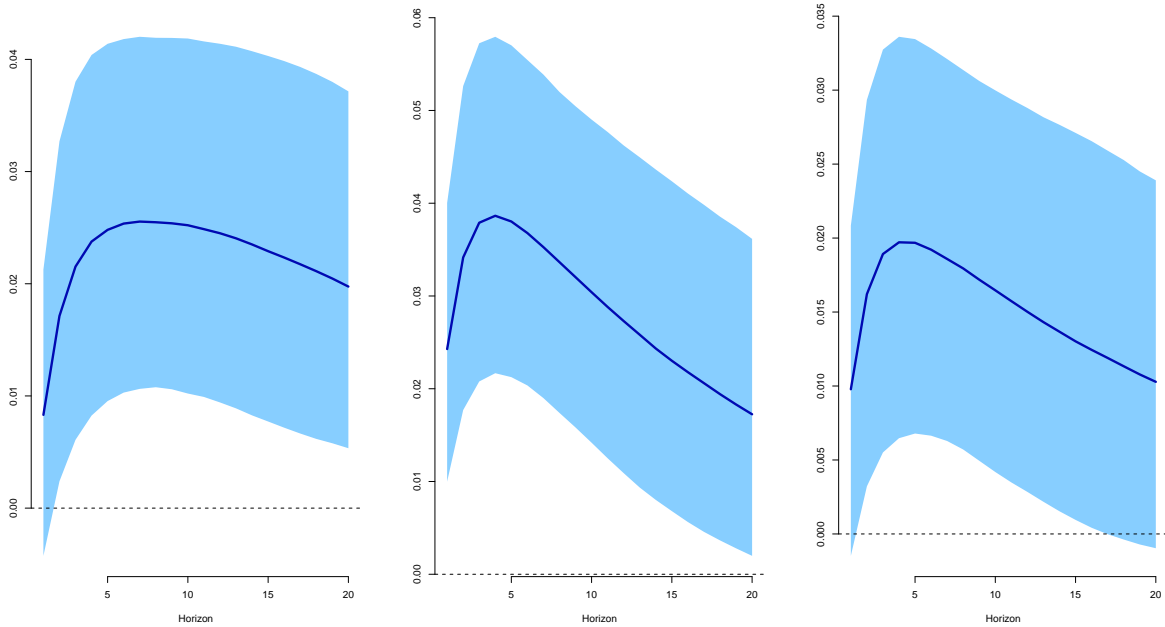


Figure 5: Response of investment rate to a shock in Tobin's q for selected dates obtained from the TVP-VAR-SV model. We report the median responses of the investment rate to a shock in Tobin's q in: 1969:Q4 (left panel), 2000:Q3 (middle panel), and 2014:Q3 (right panel). Shaded areas show the 16th and 84th percentiles.

We summarize the time-varying sensitivity of i_t to q_t in figure 7 by plotting the impulse responses over time at different quarters after the shock, thus showing the responses of i_t to a shock in q_t after one quarter, four quarters, eight quarters, and sixteen quarters. It is possible to observe that the sensitivity of i_t to q_t has changed considerably over time, mainly at longer time horizons after the shock—that is, four quarters, eight quarters, and sixteen quarters after the shock to q_t . Specifically, it decreased since the early 1960s through the early 1980s, it increased since the early 1980s through the early 2000s, and it has decreased again since then.

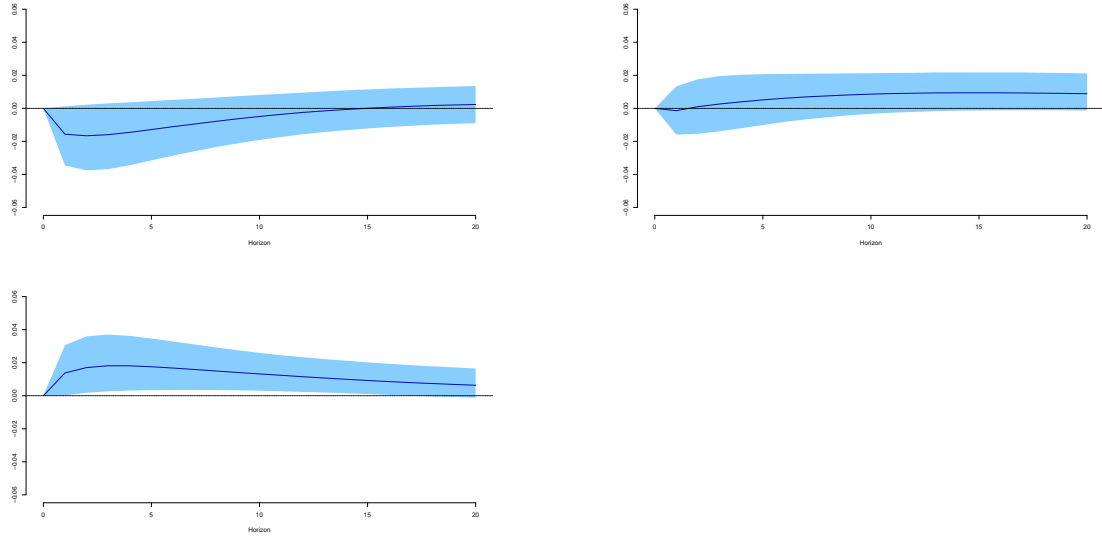


Figure 6: Differences in the response of investment rate to a shock in Tobin's q for selected dates obtained from the TVP-VAR-SV model. We report the differences of the median responses of the investment rate to a shock in Tobin's q between: 1969:Q4-2000:Q3 (top-left panel), 1969:Q4-2014:Q3 (top-right panel), and 2000:Q3-2014:Q3 (bottom-left panel). Shaded areas show the 16th and 84th percentiles.

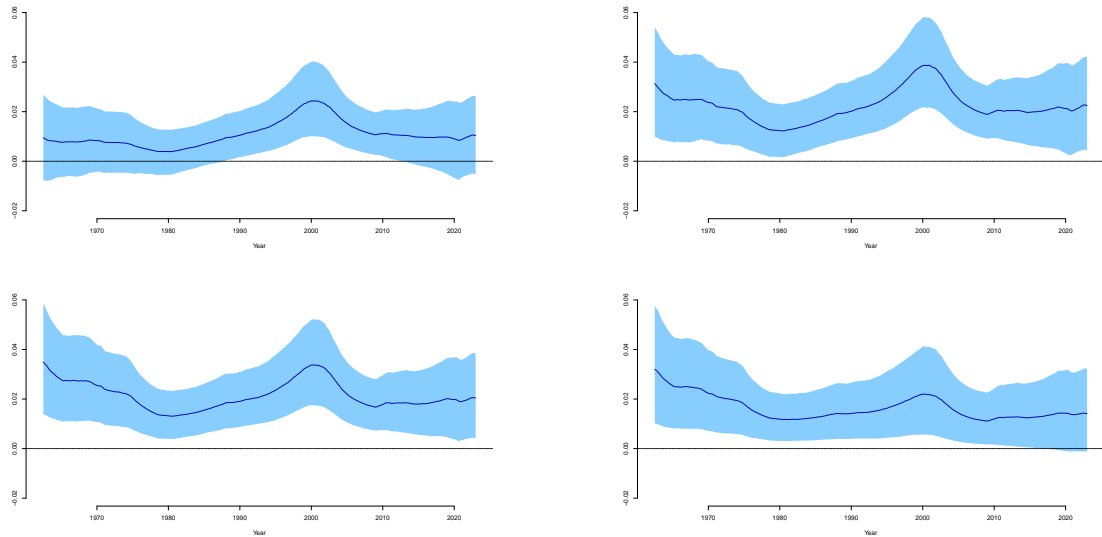


Figure 7: Response of investment rate to a shock in Tobin's q over time and at different quarters obtained from the TVP-VAR-SV model. We report the median responses of the investment rate to a shock in Tobin's q after: one quarter (top-left panel), four quarters (top-right panel), eight quarters (bottom-left panel), and sixteen quarters (bottom-right panel). Shaded areas show the 16th and 84th percentiles.

Regarding the changing nature of the effects of c_t on i_t , we first plot the impulse responses following a shock to c_t considering the same dates that we considered for shocks to q_t (that is, 1969:Q4, 2000:Q3 and 2014:Q3) in figure 8. It is clear that a positive shock in c_t always increases i_t ; however, the response of i_t to a shock in c_t is considerably larger and more persistent in

the first and last quarters (1969:Q4 and 2014:Q3). Figure 9 plots the differences between the impulse responses in 1969:Q4 and 2000:Q3, 1969:Q4 and 2014:Q3, and 2000:Q3 and 2014:Q3. Considering the estimation uncertainty summarized by the error bands, it is possible to conclude that the impulse response is significantly stronger in 1969:Q4 than in 2000:Q3 (top-left panel in figure 9) and significantly weaker in 2000:Q3 than in 2014:Q3 (bottom-left panel in figure 9).

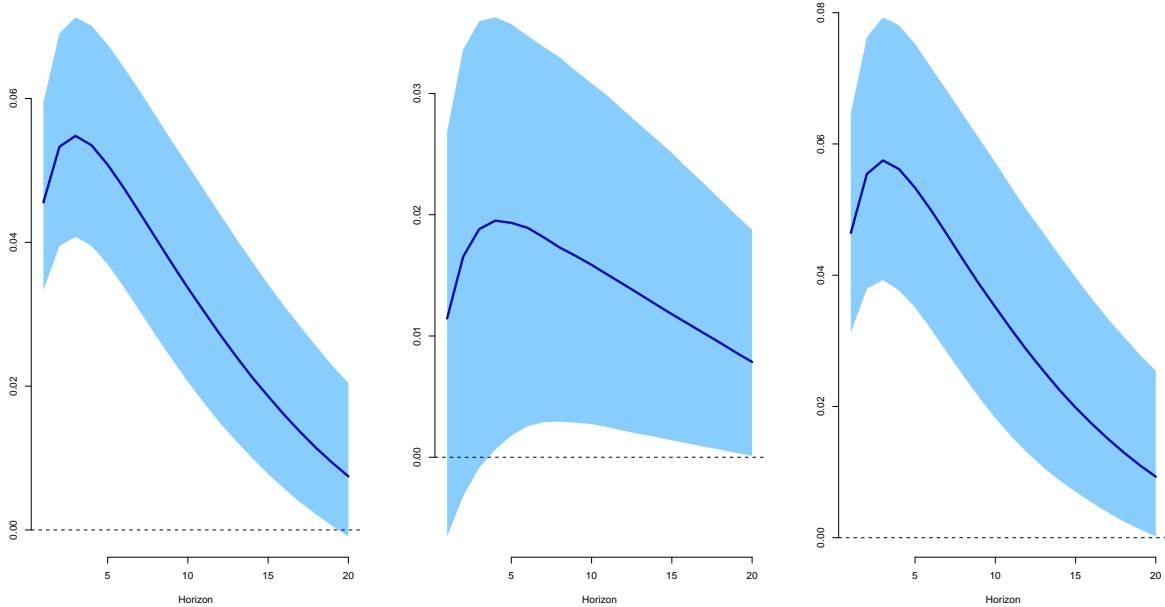


Figure 8: Response of investment rate to a shock in cash flow for selected dates obtained from the TVP-VAR-SV model. We report the median responses of the investment rate to a shock in cash flow in: 1969Q4 (left panel), 2000Q3 (middle panel), and 2014Q3 (right panel). Shaded areas show the 16th and 84th percentiles.

More importantly, we summarize the time-varying sensitivity of i_t to c_t by plotting the impulse responses of i_t to a shock in c_t over time after one quarter, four quarters, eight quarters, and sixteen quarters in figure 10. The response of i_t to a shock in c_t exhibits substantial time variation: it increased since the early 1960s through the early 1980s, it decreased since the early 1980s through the early 2000s, and it has tended to increase again since then.

The baseline results reported in this section show that the evolution of the time-varying response of i_t to a shock in c_t (figure 10) is almost the mirror image to the evolution of the time-varying response of i_t to a shock in q_t (figure 7). This indicates that the two most important investment surges in the USA experienced during the late 1960s and early 2000s were associated with different factors: a higher sensitivity of i_t to c_t and a higher sensitivity of i_t to q_t , respectively.

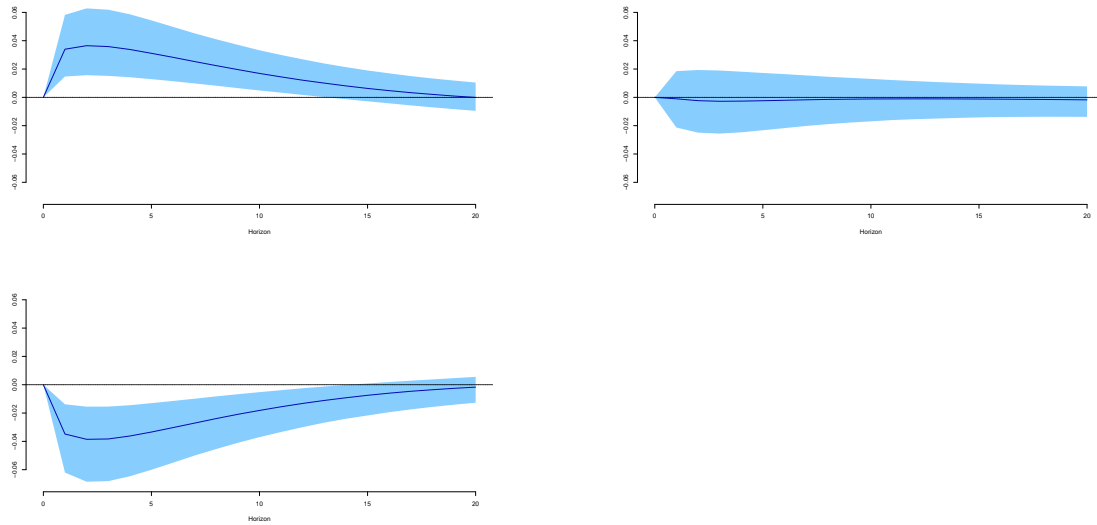


Figure 9: Differences in the response of investment rate to a shock in cash flow for selected dates obtained from the TVP-VAR-SV model. We report the differences of the median responses of the investment rate to a shock in cash flow between: 1969:Q4-2000:Q3 (top-left panel), 1969:Q4-2014:Q3 (top-right panel), and 2000:Q3-2014:Q3 (bottom-left panel). Shaded areas show the 16th and 84th percentiles.

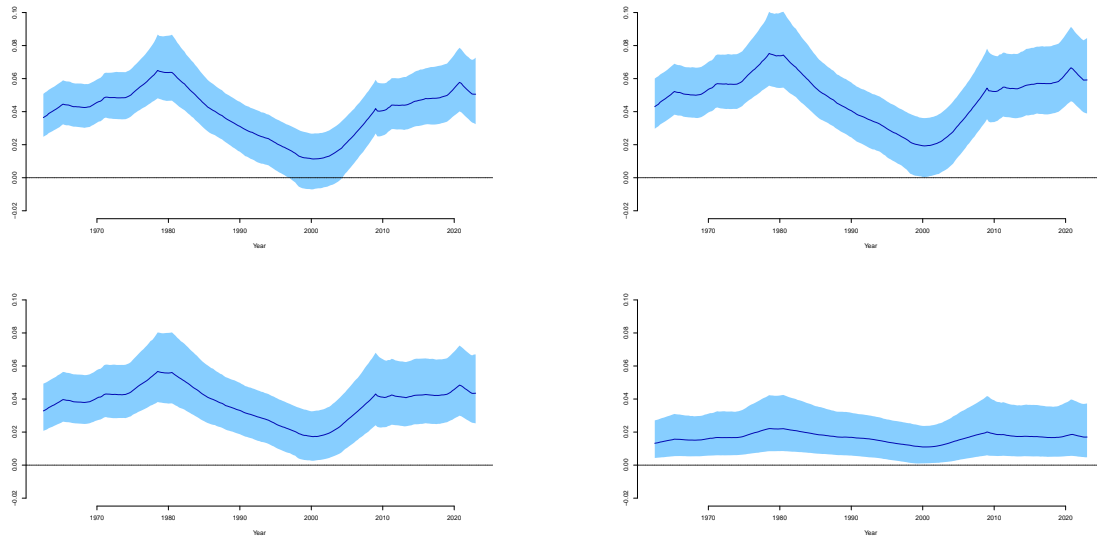


Figure 10: Response of investment rate to a shock in cash flow over time and at different quarters obtained from the TVP-VAR-SV model. We report the median responses of the investment rate to a shock in cash flow after: one quarter (top-left panel), four quarters (top-right panel), eight quarters (bottom-left panel), and sixteen quarters (bottom-right panel). Shaded areas show the 16th and 84th percentiles.

6. Robustness checks

We carried out three main robustness checks, which we summarize in this section. First, we considered an alternative Cholesky ordering of variables: $\mathbf{y}'_t = (c_t, q_t, i_t)'$, which implies that

we order c_t first, q_t second, and i_t last. Thus, we still assume that i_t is the most endogenous variable in the system since it is affected contemporaneously by both c_t and q_t ; however, c_t is now assumed to be the most exogenous variable in the system as it effects both q_t and i_t in the same period, while shocks to q_t effect only c_t within the same period.

Figures 11 and 12 show the time-varying responses of i_t to a shock to q_t and of i_t to a shock to c_t , respectively, obtained from the TVP-VAR-SV model using the alternative Cholesky ordering to generate the impulse response functions.⁸ Both figures are almost identical to the ones obtained from the baseline Cholesky ordering discussed in the previous section—that is, figure 11 is almost identical to figure 7 and figure 12 is almost identical to figure 10.

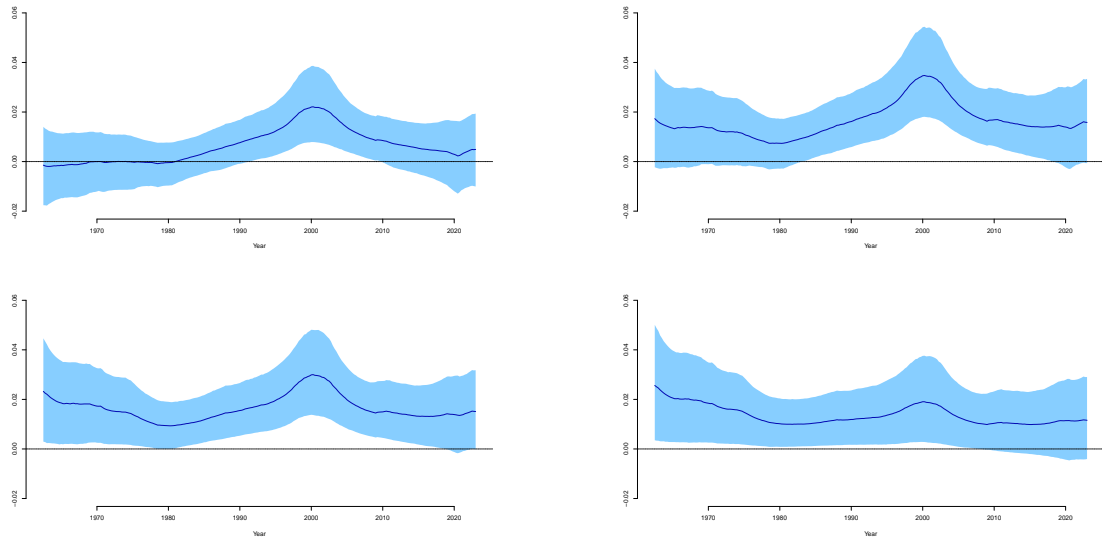


Figure 11: Response of investment rate to a shock in Tobin’s q over time and at different quarters obtained from the TVP-VAR-SV model using an alternative Cholesky ordering. We report the median responses of the investment rate to a shock in Tobin’s q after: one quarter (top-left panel), four quarters (top-right panel), eight quarters (bottom-left panel), and sixteen quarters (bottom-right panel). Shaded areas show the 16th and 84th percentiles.

Second, we explore to what extent the baseline results reported in section 5 are potentially affected by the effects derived from the most recent COVID-19 recession. Therefore, we estimate the TVP-VAR-SV model using again the baseline Cholesky ordering of variables depicted by y_t in section 5 but without considering the post-COVID-19 recession observations.⁹ The main

⁸Figures C.1 and C.2 in appendix C show the impulse responses following a shock to q_t for the same dates selected in section 5 as well as the differences across the respective dates but now considering the alternative ordering of variables; while figures C.3 and C.4 do the same but for a shock to c_t . It is possible to observe that the results are almost identical to the ones presented in the previous section, the only exception being that the response of i_t to a shock to q_t is also significantly weaker in 1969:Q4 than in 2000:Q3 (top-left panel in figure C.2) when using the alternative ordering.

⁹Figure C.5 in appendix C shows the stochastic volatility of the shocks obtained from the TVP-VAR-SV model considering the reduced sample. Overall, the evolution of the volatility of the three shocks is almost identical to the one found by the baseline results, shown in figure 4.

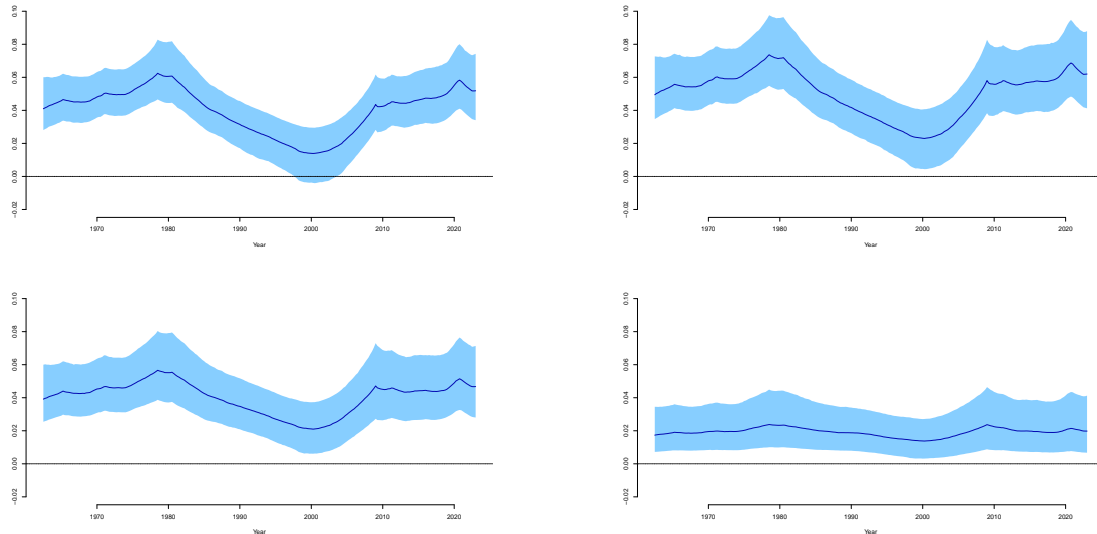


Figure 12: Response of investment rate to a shock in cash flow over time and at different quarters obtained from the TVP-VAR-SV model using an alternative Cholesky ordering. We report the median responses of the investment rate to a shock in cash flow after: one quarter (top-left panel), four quarters (top-right panel), eight quarters (bottom-left panel), and sixteen quarters (bottom-right panel). Shaded areas show the 16th and 84th percentiles.

results are shown in figures 13 and 14. The time-varying response of i_t to a shock to q_t shown in figure 13 is almost identical to the baseline results in figure 7—the only difference being the relatively wider error bands during the last ten years of the estimation in figure 13. Likewise, the time-varying response of i_t to a shock to c_t shown in figure 14 is virtually identical to the baseline results in figure 10.

Finally, we estimate the TVP-VAR-SV model without considering the post-COVID-19 recession observations and using the alternative Cholesky ordering defined by \mathbf{y}'_t in the present section. We find that the evolution of the time-varying sensitivity of i_t to a shock to q_t at different quarters after the shock, shown in figure 15, is virtually identical to the baseline results shown in figure 7; while the evolution of the time-varying response of i_t to a shock to c_t at different quarters after the shock, shown in figure 16, is also identical to the baseline results shown in figure 10.

To summarize, the robustness checks presented in this section—namely: (i) using an alternative Cholesky ordering of variables; (ii) estimating the TVP-VAR-SV model without considering the effects of the COVID-19 recession; and (iii) estimating the TVP-VAR-SV model without considering the effects of the COVID-19 recession and using an alternative ordering of variables—strongly corroborate the baseline results summarized in section 5.

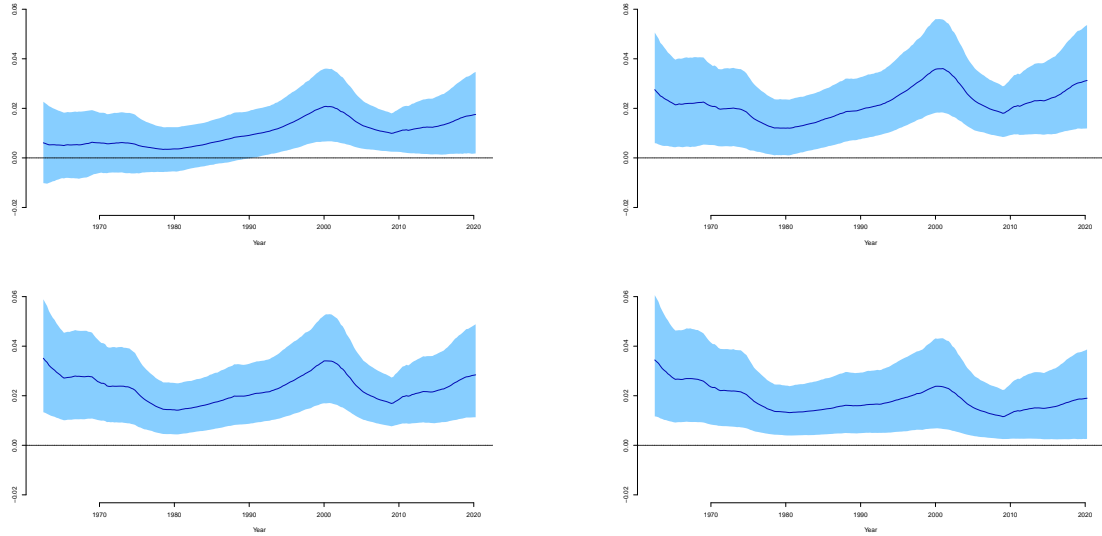


Figure 13: Response of investment rate to a shock in Tobin's q over time and at different quarters obtained from the TVP-VAR-SV model without considering the post-COVID-19 recession observations (2020:Q2-2022:Q4). We report the median responses of the investment rate to a shock in Tobin's q after: one quarter (top-left panel), four quarters (top-right panel), eight quarters (bottom-left panel), and sixteen quarters (bottom-right panel). Shaded areas show the 16th and 84th percentiles.

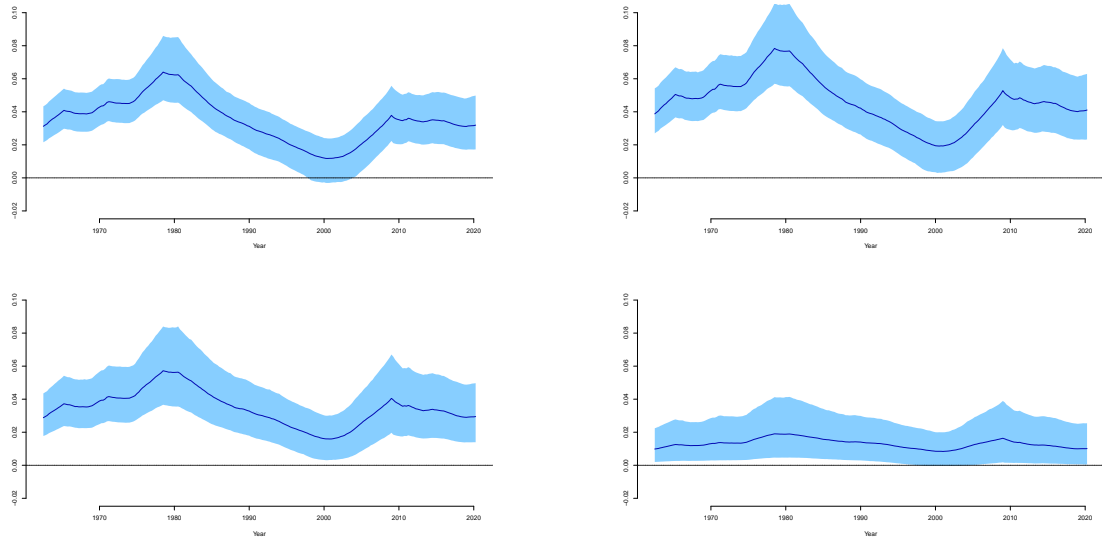


Figure 14: Response of investment rate to a shock in cash flow over time and at different quarters obtained from the TVP-VAR-SV model without considering the post-COVID-19 recession observations (2020:Q2-2022:Q4). We report the median responses of the investment rate to a shock in cash flow after: one quarter (top-left panel), four quarters (top-right panel), eight quarters (bottom-left panel), and sixteen quarters (bottom-right panel). Shaded areas show the 16th and 84th percentiles.

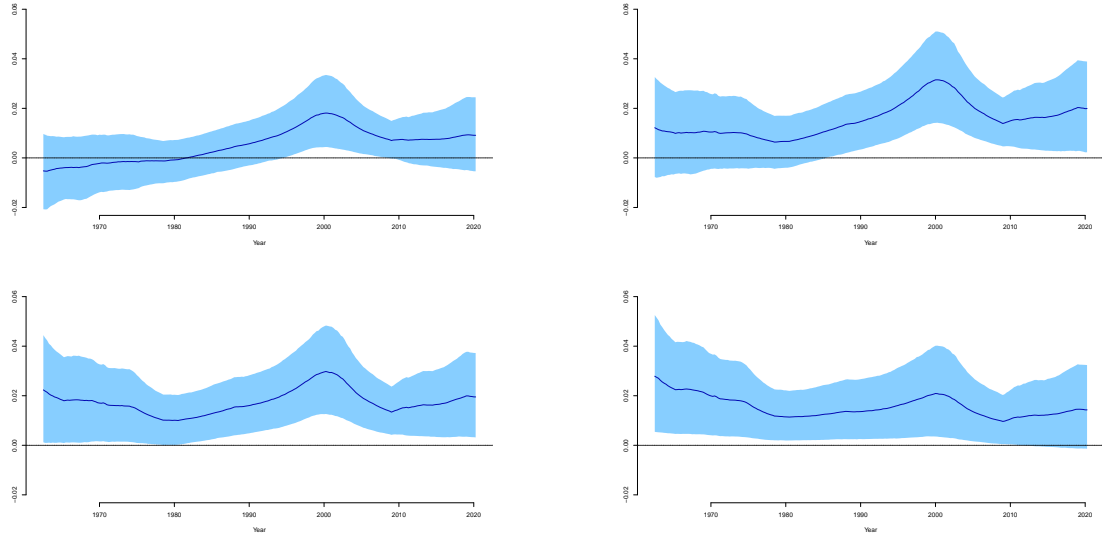


Figure 15: Response of investment rate to a shock in Tobin's q over time and at different quarters obtained from the TVP-VAR-SV model without considering the post-COVID-19 recession observations (2020:Q2-2022:Q4) and using an alternative Cholesky ordering. We report the median responses of the investment rate to a shock in Tobin's q after: one quarter (top-left panel), four quarters (top-right panel), eight quarters (bottom-left panel), and sixteen quarters (bottom-right panel). Shaded areas show the 16th and 84th percentiles.

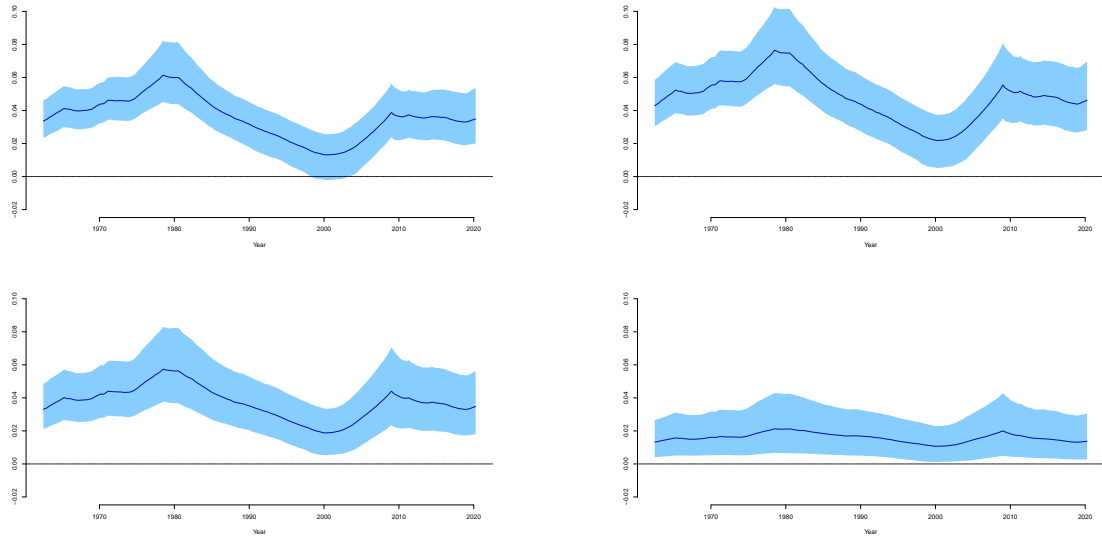


Figure 16: Response of investment rate to a shock in cash flow over time and at different quarters obtained from the TVP-VAR-SV model without considering the post-COVID-19 recession observations (2020:Q2-2022:Q4) and using an alternative Cholesky ordering. We report the median responses of the investment rate to a shock in cash flow after: one quarter (top-left panel), four quarters (top-right panel), eight quarters (bottom-left panel), and sixteen quarters (bottom-right panel). Shaded areas show the 16th and 84th percentiles.

7. Conclusions

What are the time-varying effects of the availability of external equity finance, approximated by Tobin's q , and the availability of internal funds, approximated by cash flow, on the dynamics

of aggregate investment in the USA? We answer this question by estimating a time-varying parameter vector autoregression model with stochastic volatility via Bayesian methods. We find strong empirical evidence showing that investment exhibits important time-varying sensitivities to both variables during the the post-World War II period, thus indicating the existence of relevant structural changes in the dynamics of investment and its linkages to Tobin's q and cash flow. The evolution of the time-varying sensitivity of investment to a shock to Tobin's q decreased since the early 1960s through the early 1980s, increased since the early 1980s through the early 2000s, and it has decreased importantly again since then. On the other hand, the time-varying sensitivity of investment to a shock to cash flow increased since the early 1960s through the early 1980s, decreased since the early 1980s through the early 2000s, and it has tended to increase importantly again since then. Thus, the evolution of the time-varying response of investment to shocks to Tobin's q is almost the mirror image to the evolution of the time-varying response of investment to shocks to cash flow. These results suggest that, although Tobin's q and cash flow can be regarded as complementary sources of information for investment decisions, the relative importance of each variable for investment dynamics has changed considerably over time, so that both variables should also be regarded as alternative to each other in order to understand short-run fluctuations in investment.

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Appendix

Appendix A Priors

In all the estimated TVP-VAR-SV models, $n = 3$ in \mathbf{y}_t in equation (2), which means that we considered three endogenous variables in the systems. Following Primiceri (2005), we use the following prior distributions for estimation:

$$\begin{aligned}\beta_0 &\sim \mathcal{N}\left(\widehat{\beta}_{OLS}, 4 * \widehat{V}(\widehat{\beta}_{OLS})\right), \\ \mathbf{A}_0 &\sim \mathcal{N}\left(\widehat{\mathbf{A}}_{OLS}, 4 * \widehat{V}(\widehat{\mathbf{A}}_{OLS})\right), \\ \log \sigma_0 &\sim \mathcal{N}(\log \widehat{\sigma}_{OLS}, \mathbf{I}_n), \\ \mathbf{Q} &\sim \mathcal{IW}\left(k_Q^2 * 40 * \widehat{V}(\widehat{\beta}_{OLS}), 40\right), \\ \mathbf{S}_1 &\sim \mathcal{IW}\left(k_S^2 * 2 * \widehat{V}(\widehat{\mathbf{A}}_{1,OLS}), 2\right), \\ \mathbf{S}_2 &\sim \mathcal{IW}\left(k_S^2 * 3 * \widehat{V}(\widehat{\mathbf{A}}_{2,OLS}), 3\right), \\ \mathbf{W} &\sim \mathcal{IW}\left(k_W^2 * 4 * \mathbf{I}_n, 4\right),\end{aligned}$$

where $\widehat{\beta}_{OLS}$ and $\widehat{V}(\widehat{\beta}_{OLS})$ are the mean and variance of β_0 , respectively; $\widehat{\mathbf{A}}_{OLS}$ and $\widehat{V}(\widehat{\mathbf{A}}_{OLS})$ are the mean and variance of \mathbf{A}_0 , respectively; \mathbf{S}_1 and \mathbf{S}_2 denote the two blocks of \mathbf{S} , such that $\widehat{\mathbf{A}}_{1,OLS}$ and $\widehat{\mathbf{A}}_{2,OLS}$ are the two corresponding blocks of $\widehat{\mathbf{A}}_{OLS}$; $k_Q = k_W = 0.01$; and $k_S = 0.1$.

As in Primiceri (2005), we also use the first 10 years (40 observations with quarterly data) to calibrate the prior distributions, so that $\widehat{\beta}_{OLS}$, $\widehat{V}(\widehat{\beta}_{OLS})$, $\widehat{\mathbf{A}}_{OLS}$, $\widehat{V}(\widehat{\mathbf{A}}_{OLS})$, and $\log \sigma_0$ were all obtained via training sample OLS retrieved from VAR models with constant parameters and constant variance-covariance matrices.

Appendix B Summary of the MCMC sampling algorithm

We implement the MCMC sampling algorithm of Primiceri (2005) considering the correction noted by Del Negro and Primiceri (2015), which corresponds to “algorithm 2” in the latter. Compared to Primiceri (2005)’s original algorithm, Del Negro and Primiceri (2015) propose that the sampling of the stochastic volatilities should be carried out after the sampling of the states of the mixture of normals components approximation to a $\log \chi^2(1)$ density. Since the mixture of normals is only an approximation of the $\log \chi^2(1)$ density, Del Negro and Primiceri

(2015)'s algorithm can be regarded as a sampler from an approximate posterior. However, inverting the order of the draws is extremely important as this allows for the construction of an algorithm that represents a superior approximation to the true posterior distribution compared to the original Gibbs sampler developed by Primiceri (2005).

Let us denote $\beta^T = \{\beta_t\}_{t=1}^T$, $\Sigma^T = \{\Sigma_t\}_{t=1}^T$, $\mathbf{A}^T = \{\mathbf{A}_t\}_{t=1}^T$, $\theta = (\beta^T, \Sigma^T, \mathbf{A}_t)$, and $\mathbf{X} = (\mathbf{Q}, \mathbf{S}, \mathbf{W})$. For simplicity, in what follows we omit the dependence of the conditional posteriors on the observed data as well as the variables that affect the conditional posteriors if the latter are independent of a particular block in the Gibbs sampler. Thus, the sampling scheme comprises the following steps:

1. Initialize \mathbf{A}^T , Σ^T , \mathbf{s}^T and \mathbf{X} , where $\mathbf{s}^T = \{\mathbf{s}_t\}_{t=1}^T$ corresponds to the mixture indicators (auxiliary discrete variables) that select the component of the mixture for each variable at each date.
2. Draw β^T from $p(\beta^T | \theta^{-\beta^T}, \Sigma^T)$ using the Carter and Kohn (1994) (CK) algorithm.
3. Draw \mathbf{Q} from $p(\mathbf{Q} | \beta^T)$, which corresponds to an \mathcal{IW} distribution.
4. Draw \mathbf{A}^T from $p(\mathbf{A}^T | \theta^{-\mathbf{A}^T}, \Sigma^T)$ using the CK algorithm.
5. Draw \mathbf{S} from $p(\mathbf{S}^T | \theta^{-\mathbf{S}}, \Sigma^T)$, which consists of two blocks that are \mathcal{IW} distributions.
6. Draw \mathbf{s}^T from $p(\mathbf{s}^T | \Sigma^T, \theta)$ using the Kim et al. (1998) algorithm.
7. Draw Σ^T from $p(\Sigma^T | \theta, \mathbf{s}^T)$ using the CK algorithm.
8. Draw \mathbf{W} from $p(\mathbf{W} | \Sigma^T)$, which corresponds to an \mathcal{IW} distribution.
9. Go to step 2.

Appendix C Additional results

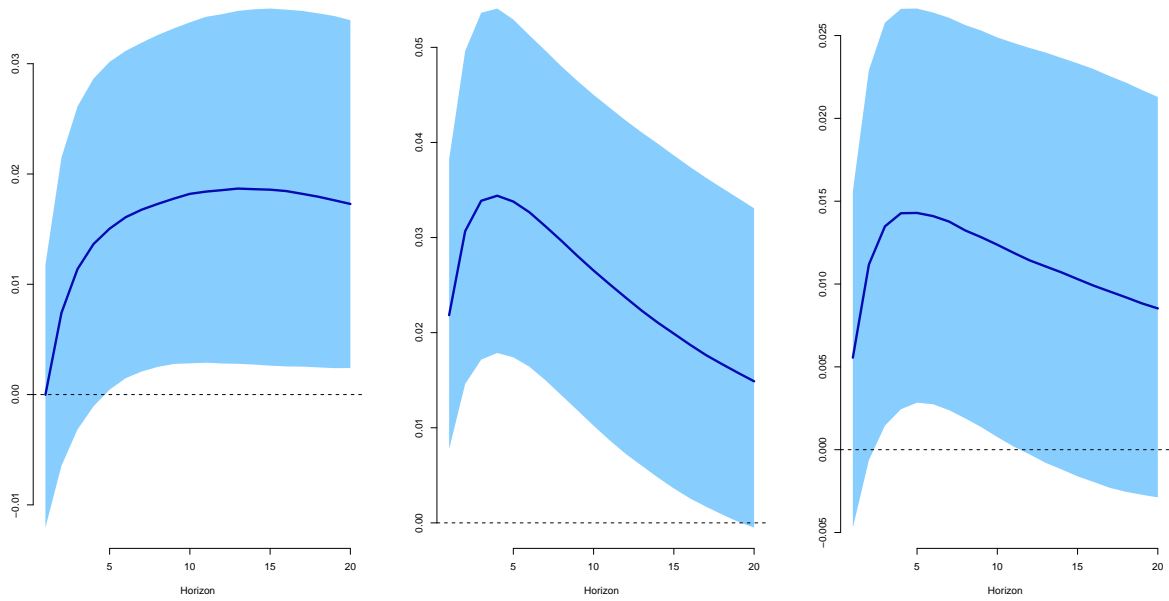


Figure C.1: Response of investment rate to a shock in Tobin's q for selected dates obtained from the TVP-VAR-SV model using an alternative Cholesky ordering. We report the median responses of the investment rate to a shock in Tobin's q in: 1969:Q4 (left panel), 2000:Q3 (middle panel), and 2014:Q3 (right panel). Shaded areas show the 16th and 84th percentiles.

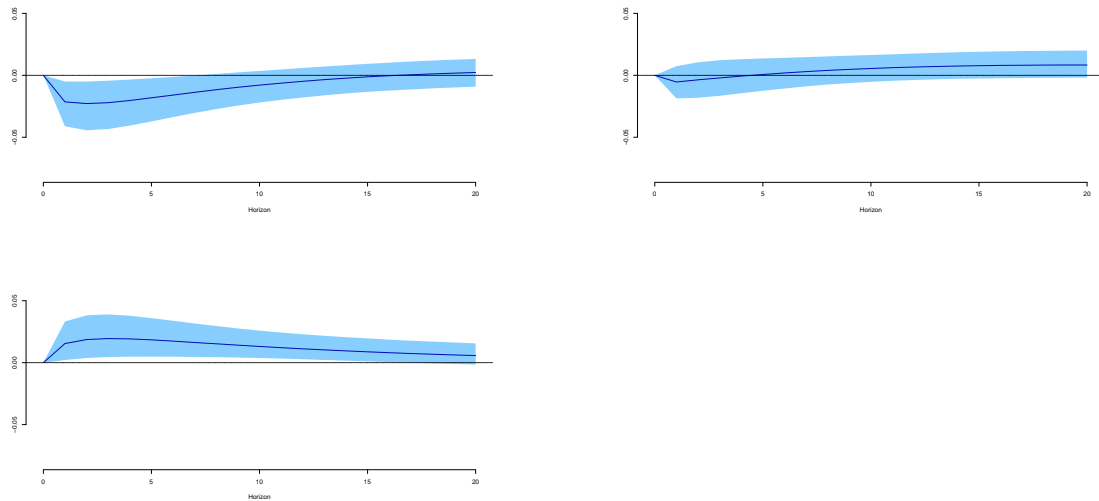


Figure C.2: Differences in the response of investment rate to a shock in Tobin's q for selected dates obtained from the TVP-VAR-SV model using an alternative Cholesky ordering. We report the differences of the median responses of the investment rate to a shock in Tobin's q between: 1969:Q4-2000:Q3 (top-left panel), 1969:Q4-2014:Q3 (top-right panel), and 2000:Q3-2014:Q3 (bottom-left panel). Shaded areas show the 16th and 84th percentiles.

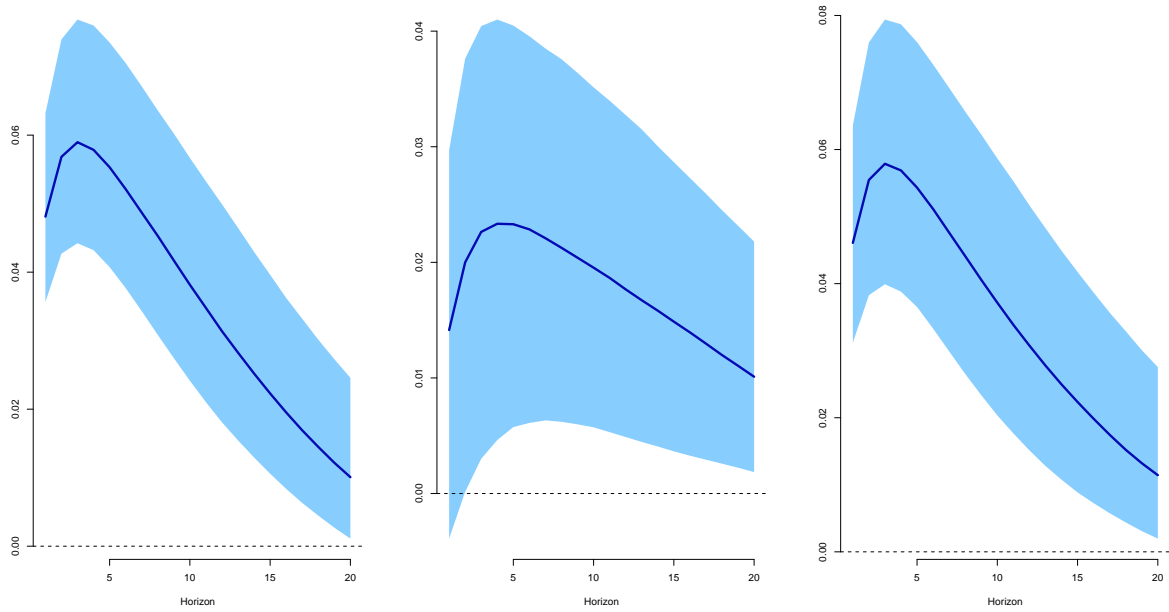


Figure C.3: Response of investment rate to a shock in cash flow for selected dates obtained from the TVP-VAR-SV model using an alternative Cholesky ordering. We report the median responses of the investment rate to a shock in cash flow in: 1969:Q4 (left panel), 2000:Q3 (middle panel), and 2014:Q3 (right panel). Shaded areas show the 16th and 84th percentiles.

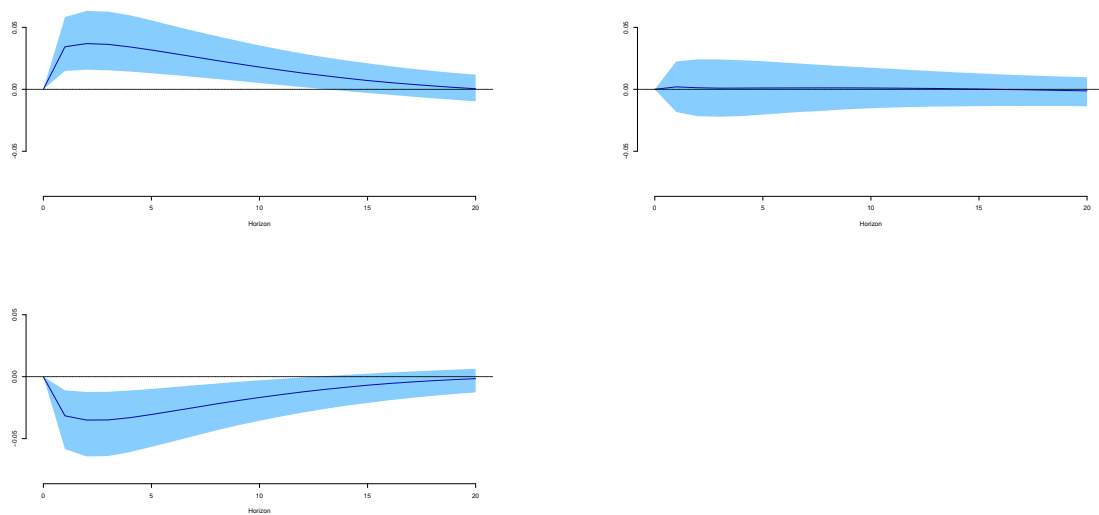


Figure C.4: Differences in the response of investment rate to a shock in cash flow for selected dates obtained from the TVP-VAR-SV model using an alternative Cholesky ordering. We report the differences of the median responses of the investment rate to a shock in cash flow between: 1969:Q4-2000:Q3 (top-left panel), 1969:Q4-2014:Q3 (top-right panel), and 2000:Q3-2014:Q3 (bottom-left panel). Shaded areas show the 16th and 84th percentiles.

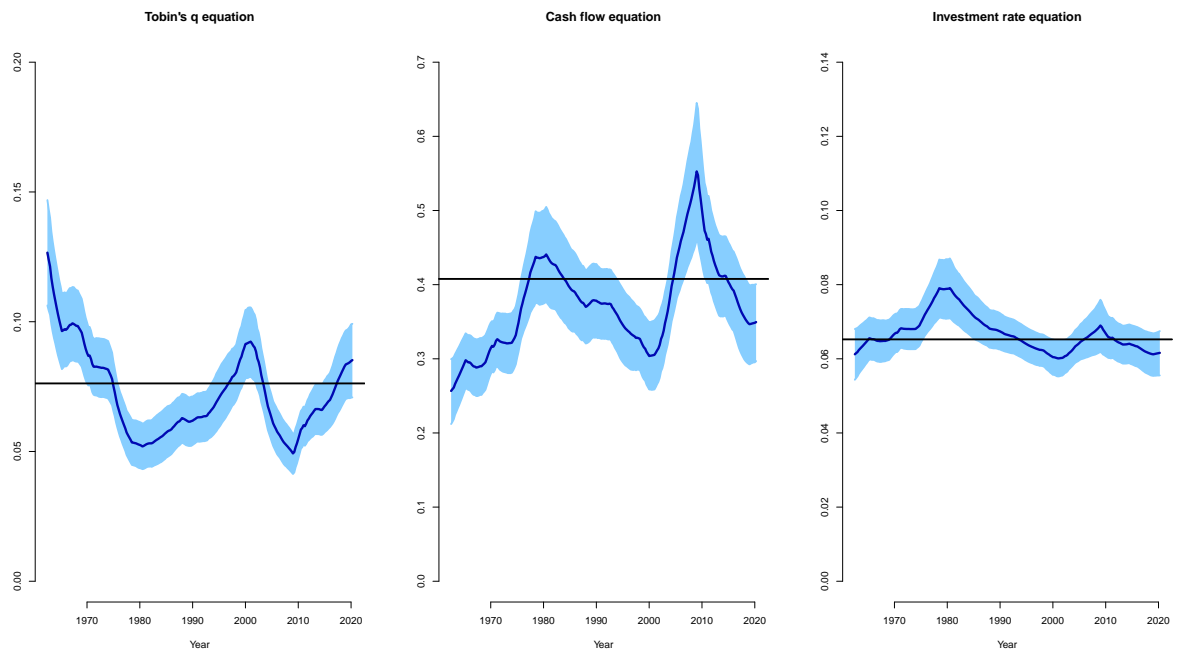


Figure C.5: Posterior means of the standard deviations of residuals obtained from the TVP-VAR-SV model without considering the post-COVID-19 recession observations (2020:Q2-2022:Q4). We report the time series plots of the means of the standard deviations of the residuals of Tobin's q equation, cash flow equation, and investment rate equation in the TVP-VAR-SV model. Shaded areas show the 16th and 84th percentiles. Black horizontal lines show to the means of the standard deviations of the residuals obtained from a standard VAR model (without TVP or SV) estimated via frequentist methods.