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Abstract

Economic analysis has approached the problem of the neutrality of money through methods of supply-demand equilibrium in which changes in aggregate demand due to monetary or fiscal policy are equivalent to changes in the denomination of the monetary standard. We re-examine this question using statistical equilibrium methods adapted from statistical physics, which address both the central tendency of prices in equilibrium and the systematic fluctuation of prices around the central tendency. From this perspective the neutrality of money in the sense of the invariance of real economic outcomes to aggregate demand shocks depends on the adjustment of both expectations of the average level of wages and prices and the further adjustment of anticipations of the scale of fluctuations in prices and wage offers. We illustrate these conclusions through a model of wage and employment outcomes in a labor market model comprised of informationally constrained workers and employers whose interactions have a non-zero impact on wages. The model endogenizes employment interactions between workers and employers in terms of a quantal response equilibrium and produces an equilibrium level of unemployment as a statistical feature of a decentralized labor market. Shocks to the economy can produce short-run increases in involuntary unemployment arising from inertia in the adjustment of expectations. Even after agents align their expectations with market outcomes, unless they also adjust their expectations of the scale of statistical fluctuations in wages, a negative shock to demand can result in higher levels of equilibrium unemployment. In this way the model exhibits a particular type of non-neutrality of money in the short-run and long-run.

Keywords: Neutrality of money, Wage distribution, Labor market, Involuntary unemployment, Statistical equilibrium
JEL Classification: C18, D80, E10, E24, E70

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1 Introduction

The classic analysis of the neutrality of money studies this problem in the context of supply-demand equilibrium modeling that assumes a single price in each market. In this context an aggregate demand shock due to, for example, a change in monetary policy is formally equivalent to a change in the denomination of the currency and has no impact on market equilibrium. We reconsider this conclusion using statistical models of market equilibrium adapted from the methods of statistical physics. This analysis reveals that adjustment of expectations of the central tendency of money prices is not sufficient to guarantee the invariance of market outcomes to an aggregate demand shock. The full restoration of pre-shock levels of real wages and unemployment also requires the adjustment of expectations of the scale of fluctuation of price and wage offers to the shock. It is only when both of these behavioral parameters have adjusted that an aggregate demand shock is equivalent to a change in the denomination of the currency and money is fully neutral.

Statistical equilibrium models prices in a market (wages in the labor market) as a non-degenerate frequency distribution around a central tendency. Because of the statistical variability of wage and price offers, the responses of agents such as workers and employers to particular offers has a random component as in the widely used quantal response models of individual behavior. Statistical equilibrium is conceived as a stochastic process where the number of agents in any state remains constant due to a balance between the random processes moving agents into and out of each state. Even in statistical equilibrium individual agents are moving ceaselessly into and out of the available states, for example, from employment to unemployment. Statistical equilibrium is the appropriate conceptual tool for investigating such phenomena as the neutrality of money as well as frictional and involuntary unemployment. The pioneering work of E. T. Jaynes [Jaynes, 1957] establishes that a powerful and simple characterization of statistical equilibrium in general systems is the maximization of Shannon informational entropy subject to constraints that describe the structure of the system. In the context of economic markets, these constraints reflect the behavioral and institutional regularities shaping the market.

2 Statistical equilibrium in the labor market

Labor market interactions between workers and employers determine aggregate employment and wage outcomes that can be understood in terms of statistical equilibrium frequency distributions [Foley, 1994, 1996]. When workers and employers face information-processing constraints their actions are described probabilistically by logit-quantal response distributions [Luce, 1959; McKelvey and Palfrey, 1995], which are defined by agents' expectations of the average level and scale of fluctuations of wage offers. Quantal responses in actions induce strong correlations between wages and employment that lead to a non-degenerate wage distribution and persistent unemployment in statistical equilibrium. When workers' and employers' hiring interactions primarily depend on the wage, the labor market can be represented by a joint distribution over each agent's actions and the wage level. Statistical equilibrium is the joint distribution that maximizes informational entropy subject to the behavioral and institutional constraints of competitive labor market interactions [Scharfenaker and Foley, 2017; Scharfenaker, 2020a,b]. In supply-demand models the price (or wage) can be represented mathematically by a single-valued variable, but in statistical equilibrium the price must be represented by a normalized frequency distribution, which determines both its mean level and the range of fluctuations.

In a statistical equilibrium where agents' expectations about the labor market, which can include the average level and scale of fluctuation of the wage as well as expectations about average unemployment and job vacancies, endogenous fluctuations in wages produce persistent unemployment as a consequence of decentralized labor market interactions and agents' information-processing constraints. When shocks are permanent, agents' expectations of average wages can adjust, leading to a new statistical equilibrium but at levels of unemployment that may be above or below those prior to the shock, depending on the separate adjustment of agents' expectations of the scale of fluctuations of wage offers. When individual behavior is modeled probabilistically in statistical equilibrium the endogenous fluctuations of agents into different employment and wage states elucidates the concept of frictional unemployment while also implying the novel concept of frictional employment.

Another important implication of our model is that changes in a nominal exogenous variable due, for example, to a change in monetary policy or aggregate demand, can result in the uneven adjustment of agents' expectations leading to real changes in the wage and level of unemployment. While Walrasian equilibrium level of unemployment is the level that remains after the "grinding out" of such changes to the broader economic environment, statistical equilibrium unemployment "builds in" these changes due to the slow and uneven adjustment of expectations to the fluctuations in prices. In these situations a shock to aggregate demand is not the same as a change in the denomination of the currency, and money in this sense is no longer necessarily neutral.

3 Entropy Constrained Behavior

Following Scharfenaker and Foley [2017]; Scharfenaker [2020b]; Foley [2020a], as well as Sims [2003]; Matějka and McKay [2015], we adopt an information theoretic form of bounded rationality and model workers and employers as facing a decision problem of choosing an action from a finite set of actions $A \in \mathcal{A}$ conditional on a payoff $u[A, \omega] : \mathcal{A} \to \mathbb{R}$ and mixed strategy $f[A|\omega] : \mathcal{A} \times \mathbb{R} \to (0, 1)$ which is a function of the money wage $\omega \in \mathbb{R}_{\geq 0}$. Given the payoff of each type of agent for choosing an action, there is a mixed strategy that maximizes expected payoff subject to a minimum constraint on the informational entropy of the mixed strategy, which implies rational inattention behavior [Sims, 2003]. As shown in Scharfenaker and Foley [2017] the entropy-constrained payoff-maximizing mixed strategy can also be viewed as maximizing the entropy of the mixed strategy distribution subject to a minimum constraint on expected payoff, a dual formulation that implies satisficing bounded rationality [Simon, 1956]. In both cases the frequency of actions conditional on the payoff takes the Gibbs form:

$$f[A|\omega] = \frac{e^{\frac{u[A,\omega]}{T}}}{\sum_{\mathcal{A}} e^{\frac{u[A,\omega]}{T}}} \tag{1}$$

With two actions $\mathcal{A} = \{a, \bar{a}\}$, the Gibbs distribution reduces to the logistic quantal response function expressed as a difference in payoffs:

$$f[a|x] = \frac{e^{\frac{u[a,\omega]}{T}}}{e^{\frac{u[a,\omega]}{T}} + e^{\frac{u[\bar{a},\omega]}{T}}} = \frac{1}{1 + e^{-\frac{u[a,\omega] - u[\bar{a},\omega]}{T}}} = \frac{1}{1 + e^{-\frac{\Delta \mathrm{u}[A,\omega]}{T}}} \tag{2}$$

$$f[\bar{a}|\,\omega] = 1 - f[a|\omega] = \frac{1}{1 + e^{\frac{\Delta \mathrm{u}[A,\omega]}{T}}} \tag{3}$$

These behavioral functions are characterized by the parameter T, measured in the same units as payoff, $u[a, \omega]$, which represents the scale of "just noticeable differences" in payoffs to which individual behavior responds.[1] We refer to T as the behavioral temperature due to the similar mathematical expression in thermodynamics. This parameter defines the slope of the logit quantal response function and thus the scale of fluctuations of individual behavior.

An intuitive way of understanding entropy constrained behavior and Eq. 1 is in terms of the exploration-exploitation tradeoff used in reinforcement learning [Schwartenbeck et al., 2013]. When an agent faces a problem of maximizing expected utility in a complex decision-

[1]See Appendix A for proofs.

making environment pursuing actions that maximize the value of expected utility corresponds to exploitation of the environment whereas exploration corresponds to visiting or sampling alternative states. The behavioral temperature T captures the informational tradeoff associated with exploitation and exploration of complex or "rugged" decision landscapes. While traditional economic interpretations of a positive decision temperature imply deviations from optimal solutions, the element of partial randomization in decision making has been shown in the development of AI, among other fields, to be an essential feature of decision making and optimal search algorithms and not a bug [Miller, 2016].

4 The "Shape-Up" Labor Market

In the simplest treatment of a labor market, all interactions occur once per unit time interval. Peter Doeringer and Michael Piore [Doeringer and Piore, 1971] refer to this setting as a *shape-up labor market*. In the shape-up economy there is a pool of unemployed workers who contract their labor for a fixed amount of time, for example one day, for a wage ω; at the end of the day the worker returns to the original unemployed state. Employers choose to either offer employment for a given wage, or to not make an offer, in which case the work is put off for that time period. Workers can either accept the job at the offered wage, in which case they are employed for that time period and work is done, or not accept the job and remain unemployed. In each period of time the same interaction between workers and employers at each money wage level repeats. The number of workers and employers at each money wage level is described by a frequency distribution over money wage levels that describes the state of the market. In the statistical equilibrium framework this frequency distribution is endogenous and determined by the statistical equilibrium condition as the frequency distribution that maximizes informational entropy subject to the constraints describing individual behavior and institutions that define the economic setting.

4.1 Workers in the Shape-Up Labor Market

We can model the typical worker's action set in the shape-up economy as either *accepting* or *turning down* an offer of employment for a given wage ω: $\mathcal{A}_w = \{a_w, \bar{a}_w\} = \{\text{accept}, \text{turn down}\}$. The payoff functions for workers in the shape-up economy differ by each action. When a worker accepts a job their payoff is the money wage they receive, ω, minus the costs of working and finding a job m_w. The payoff for turning down a job is the workers fallback position z_w.

$$u_w\left[a_w, \omega\right] = \omega - m_w \tag{4}$$

$$u_w\left[\bar{a}_w, \omega\right] = z_w \tag{5}$$

$$\Delta u_w\left[A_w, \omega\right] = \omega - (m_w + z_w) = \omega - \mu_w \tag{6}$$

We refer to the total cost of not working $\mu_w = m_w + z_w$ as the "indifference wage." With these money-equivalent payoffs workers' quantal response distributions become:

$$f_w\left[a_w \middle| \omega\right] = \frac{1}{1 + e^{-\frac{\Delta u_w[A_w,\omega]}{T_w}}} = \frac{1}{1 + e^{-\frac{\omega - \mu_w}{T_w}}} \tag{7}$$

$$f_w\left[\bar{a}_w \middle| \omega\right] = 1 - f_w\left[a_w \middle| \omega\right] = \frac{1}{1 + e^{\frac{\omega - \mu_w}{T_w}}} \tag{8}$$

In this setting the log odds of a worker accepting an offer conditional on the wage are equal to the difference in payoffs scaled by T:

$$\log\left[\frac{f_w\left[a_w \middle| \omega\right]}{f_w\left[\bar{a}_w \middle| \omega\right]}\right] = \frac{\Delta u_w\left[A_w, \omega\right]}{T_w} = \frac{\omega - \mu_w}{T_w} \tag{9}$$

These equations tell us that the probability that a worker accepts a job offer is conditional on the difference between the offered wage ω and the indifference wage μ_w. While μ_w might be understood conventionally as a "reservation wage" in the quantal response context it represents the wage at which a worker accepts a job with a probability of 50%. Only in the limit as $T \to 0$ will $f_w[a_w|\omega] = \theta[\omega - \mu_w]$, where θ is the Heaviside step function, and μ_w correspond to the reservation wage above which workers accept employment with certainty.

4.2 Employers in the Shape-Up Economy

Employers in the shape-up economy face the quantal decision to *offer* or *not offer* employment for a given wage ω, which we can model as the action set $\mathcal{A}_c = \{a_c, \bar{a}_c\} = \{\text{offer}, \text{not offer}\}$. For employers the payoff is the difference between the marginal revenue product they receive from the worker, r_c, minus the cost of the worker, ω, and any other hiring costs, such as search costs, m_c. If an employer fails to hire their fallback position is

z_c.

$$\begin{aligned} u_c\left[a_c,\omega\right] &= r_c - \omega - m_c && (10)\\ u_c\left[\bar{a}_c,\omega\right] &= z_c && (11)\\ \Delta u_c\left[A_c,\omega\right] &= -\omega + r_c - m_c - z_c = -\omega - \mu_c && (12) \end{aligned}$$

The total non-wage costs to the employer define the employer's indifference wage $\mu_c = m_c + z_c - r_c$. With these money-equivalent payoffs the conditional frequencies defining employers' actions are:

$$f_c\left[a_c|\,\omega\right] = \frac{1}{1+e^{-\frac{\Delta u_c[A_c,\omega]}{T_c}}} = \frac{1}{1+e^{\frac{\omega-\mu_c}{T_c}}} \quad (13)$$

$$f_c\left[\bar{a}_c|\,\omega\right] = \frac{1}{1+e^{-\frac{\omega-\mu_c}{T_c}}} \quad (14)$$

The log odds of an employer offering employment conditional on the wage are declining with the costs of employment and increasing with the marginal revenue product:

$$\log\left[\frac{f_c\left[a_c|\,\omega\right]}{f_c\left[\bar{a}_c|\,\omega\right]}\right] = \frac{\Delta u_c\left[A_c,\omega\right]}{T_c} = -\frac{\omega-\mu_c}{T_c} \quad (15)$$

4.3 Transaction Frequencies

The statistical equilibrium labor market model defines four dimensions of labor market interactions through conditional transaction probabilities. We assume the decisions to *accept* and *turn down* for workers and *offer* and *not offer* for employers are exclusive for each type of agent. If workers and employers decisions are only conditionally dependent on each other through the wage, we can summarize these interaction event probabilities in a joint frequency matrix for any wage ω, as in Table 1.

An employment transaction occurs when an employer offers to hire a worker at some wage and the worker accepts the offer. Workers cannot hire themselves nor can employers produce without workers. Because each agent only controls one side of the interaction the product of the conditional action frequencies $f_c[\text{offer}|\omega]$ and $f_w[\text{accept}|\omega]$ is the probability of an employment transaction at a given wage.

$$\tau[\omega] = f[a_c, a_w|\omega] = f_c[a_c|\omega]f_w[a_w|\omega] = \frac{1}{\left(1+e^{-\frac{\omega-\mu_c}{T_c}}\right)\left(1+e^{-\frac{\omega-\mu_w}{T_w}}\right)} \quad (16)$$

	Offer	No Offer	Total
Accept	$f_w\left[a_w\middle\vert\,\omega\right] f_c\left[a_c\middle\vert\,\omega\right]$	$f_w\left[a_w\middle\vert\,\omega\right] f_c\left[\bar{a}_c\middle\vert\,\omega\right]$	$\frac{1}{1+e^{-\frac{\omega-\mu_w}{T_w}}}$
Turn Down	$f_w\left[\bar{a}_w\middle\vert\,\omega\right] f_c\left[a_c\middle\vert\,\omega\right]$	$f_w\left[\bar{a}_w\middle\vert\,\omega\right] f_c\left[\bar{a}_c\middle\vert\,\omega\right]$	$\frac{1}{1+e^{\frac{\omega-\mu_w}{T_w}}}$
Total	$\frac{1}{1+e^{\frac{\omega-\mu_c}{T_c}}}$	$\frac{1}{1+e^{-\frac{\omega-\mu_c}{T_c}}}$	1

Table 1: Event space of labor-market transactions with entropy-constrained behavior. Offer/Accept defines voluntary employment, Offer/Turn Down defines voluntary unemployment, No Offer/Accept defines involuntary unemployment, and No Offer/Turn Down has no economically meaningful outcome.

We assume that employment transactions are independent of the initiation of the interaction, that is whether workers offer employment and employers accept or vice versa.[2] The events in which an employer offers but a worker turns down a job defines voluntary unemployment, whereas the event of a worker willing to accept a job at a given wage but no employer offering at that wage defines involuntary unemployment. The event in which an employer does not offer and a worker turns down is not economically meaningful in that it contributes to neither unemployment or employment interactions. We explore these concepts in more detail below.

Figure 1 shows the logit quantal response curves for a worker and employer and the employment transaction probability as a function of the wage ω.

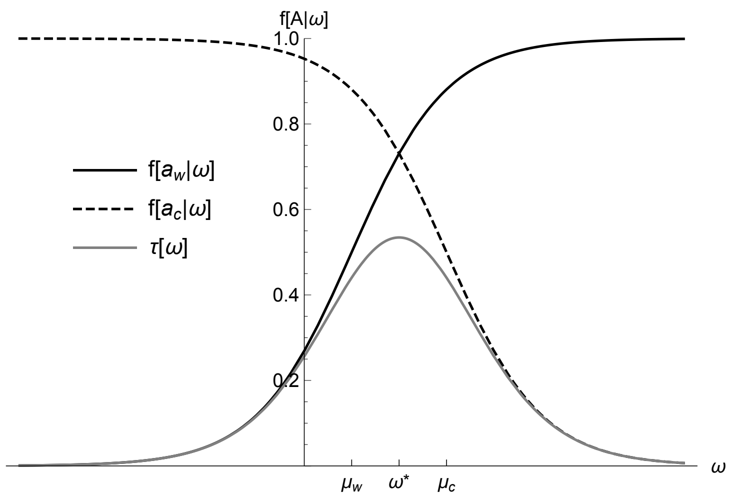

Figure 1: Logit quantal response curves for workers and employers and transaction frequencies for $\mu_w = 0.5, \mu_c = 1.5$ and $T_w = T_c = 0.5$. The mean wage conditional on employment transactions is the average of the workers' and employers' indifference wages $\bar{\omega} = 1$.

[2]This distinction can be important in the wage bargaining process, such as when only one agent in a transaction has "take-it-or-leave-it" power. In the symmetric model both agents have "take-it-or-leave-it" power.

The parameters μ_w and μ_c are the indifference points at which a typical worker would accept a job with a 50% probability and a typical employer will make an offer with a 50% probability. The difference $\mu_c - \mu_w$ can be understood as the bid-ask spread in employment transactions, which represents the opportunity for mutually advantageous transactions and the realization of economic surplus. When $\mu_w = \mu_c$ all employment transactions will be spontaneous "noise-transactions" that result is the probabilistic transfer of economic surplus from agents with low to high behavioral temperatures [Foley, 2020a].

The wage at which the quantal response frequencies of workers and employers are equal is the sum of these "indifference prices" weighted by the relative behavioral temperatures:

$$\omega^* = \mu_c \frac{T_w}{T_c + T_w} + \mu_w \frac{T_c}{T_c + T_w} \tag{17}$$

At the intersection of the labor offer and employment offer curves the transaction frequency is:

$$\tau\left[\omega^*\right] = \frac{1}{\left(1 + e^{\frac{\mu_w - \mu_c}{T_c + T_w}}\right)^2} \tag{18}$$

In the special case when both agents have identical behavioral parameters $T_w = T_c$, the transaction frequency distribution is symmetric and the wage at the average transaction frequency is $\omega^* = \frac{\mu_c + \mu_w}{2}$. These conditions approximate a supply-demand equilibrium where the equilibrium transaction wage ω^* is the average of the two agents' indifference wages and the surplus from the transaction is split evenly. When ω^* is equal to the mean wage, $\bar{\omega}$, agents' expectations are fulfilled in the sense that employers and workers are on average transacting at the mean wage.

Unlike classical search theory [Diamond, 1982], the bid-ask spread with positive behavioral temperatures does not represent the limits of the bargaining process between workers and employers since both agents can transact at any wage. Only in the limit when $T \to 0$ will the difference in indifference wages define a "frictionless" rectangle of employment interactions with area equal to the bid-ask spread $\mu_c - \mu_w$ as shown in Figure 2. In this special zero-entropy case employers will offer work with a probability equal to unity if the wage is below their reservation wage μ_c and will offer with a probability zero for a wage above μ_c because their behavior is described by the Heaviside step function $f_c[a_c|\omega] = \theta[\mu_c - \omega]$. Symmetrically, workers' behavior in the zero-entropy case is described by $f_w[a_w|\omega] = \theta[\omega - \mu_w]$, in which they will accept an offer with probability one if the wage is above their reservation wage μ_w and will never accept if the wage is below μ_w. We show below that this scenario provides a useful counterfactual benchmark for quantifying frictional unemployment and frictional employment.

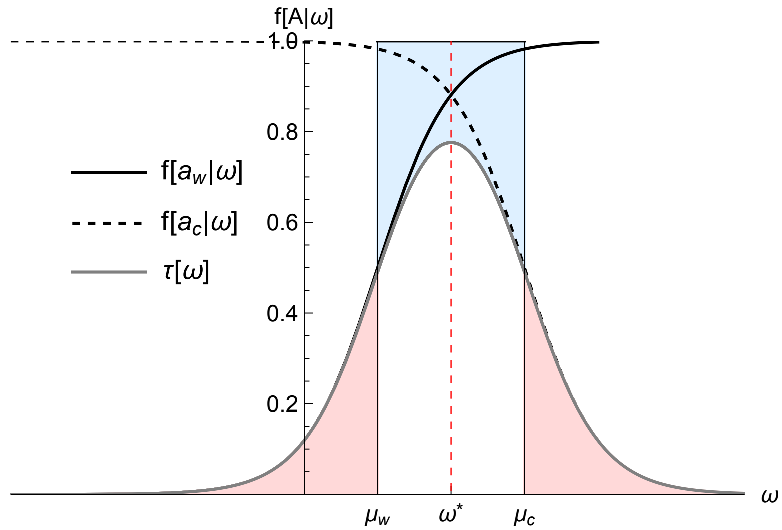

Figure 2: The space of employment transactions for $\mu_w = 0.5, \mu_c = 1.5$ and $T_w = T_c = 0.25$. The limiting case where $T \to 0$ defines a frictionless rectangle as the difference between $\theta[\mu_c - \omega]$ and $\theta[\omega - \mu_w]$ with an area equal to the bid-ask spread $\mu_c - \mu_w$. The red shaded area is the space of frictional employment transactions indicating some workers are accepting jobs below their reservation wage and some employers are offering employment above their reservation wage. Symmetrically, the blue shaded region defines the space of frictional unemployment since workers are turning down offers above their reservation wage with probability $f_w[a_w|\omega] < 1$ and employers are not offering employment below their reservation wage with probability $f_c[a_c|\omega] < 1$.

Figure 2 contrasts the frictionless transaction rectangle (corresponding to the limiting zero-entropy case $T_w = T_c = 0$) with the positive-entropy transaction probability density $\tau[\omega; T_w > 0, T_c > 0]$. This figure demonstrates that due to the positive behavioral temperatures of interacting agents some transactions that are mutually advantageous will not occur (the blue shaded area within the rectangle) while at the same time some disadvantageous transactions will occur (the red shaded area outside of the rectangle and under the transaction curve $\tau[\omega]$). The frictionless rectangle corresponds of conventional labor market interactions acts as a useful counterfactual to entropy constrained interactions. Figure 2 also shows how frictional unemployment and frictional employment are endogenous to positive-entropy labor market interactions and as discussed below, can be measured relative to the counterfactual zero-entropy case.

5 Unemployment and Vacancies with Exogenous Stochastic Job Tenure

Unemployment in the shape-up economy consists of all workers who are not hired in a period. While the shape-up economy is a good description of the type of informal markets that exist in the parking lots of Home Depot, the broader *job economy* is comprised of workers with both definite and indefinite job tenure. A job economy, however, can be described by the model of a shape-up economy if we assume job tenure is a stochastic exogenous variable that is independent of the wage.

The frequency with which a worker will see an offer of employment at any wage will depend on the number of employers per worker, and the average number of vacancies each employer attempts to fill by making job offers each period. Assuming the probability of job separation in any period, p, is exogenous, then $t = 1/p$ is the average job tenure.

The total number of jobs is $K = V + F$ where V is the number of vacant job openings and F is the number of filled jobs. There are M employers and a labor force $L = N + U$ comprised of N employed workers and U workers available to work. The number of openings relative to the total number of jobs is the job vacancy rate $v = V/K$. The ratio of unemployed workers to the labor force is the unemployment rate, $u = U/L$, while $n = N/L$ is the employment rate, and $k = K/L$ defines the job/worker ratio. The number of filled jobs is equal to the number of employed workers which implies the standard identities:

$$K - V = F = N = L - U \tag{19}$$

$$(k - v) = n = (1 - u) \tag{20}$$

$$(1 - v)k = (1 - u) \tag{21}$$

Vacancies and unemployment increase as employed workers are separated from their job, which happens at a constant rate of p, so that the number of workers becoming unemployed is pN. The change in unemployment decreases when unemployed workers find employment, which is the probability of a worker accepting employment conditional on the wage times the probability of an employer offering employment conditional on the wage weighted by the frequency of workers for any given wage, as is expressed in the observable wage distribution,

$\int \tau[\omega]f[\omega]d\,\omega$.

$$\Delta U = pN - U \int \tau[\omega]f[\omega]d\,\omega \tag{22}$$

$$\Delta U = p(1-u)L - uL \int \tau[\omega]f[\omega]d\,\omega \tag{23}$$

In equilibrium $\Delta U = 0$ and the unemployment rate is

$$u = \frac{p}{p + \int \tau[\omega]f[\omega]d\,\omega} \tag{24}$$

Similarly, the change in job vacancies increases with job separations, pN and decreases by the number of workers who find employment with probability $\tau[\omega]$ at wage $f[\omega]$.

$$\Delta V = pN - V \int \tau[\omega]f[\omega]d\,\omega \tag{25}$$

$$\Delta V = p(1-v)kL - vL \int \tau[\omega]f[\omega]d\,\omega \tag{26}$$

In equilibrium $\Delta V = 0$ and the job vacancy rate is

$$v = \frac{kp}{kp + \int \tau[\omega]f[\omega]d\,\omega} \tag{27}$$

In the United States, the job to worker ratio k has tended to be close to but below unity and the average job tenure is between close to four years, or approximately 48 months, making $p = 1/48$. Post-pandemic estimates of labor-market tightness have k closer to two.

6 The Statistical Equilibrium Wage Distribution

The assumption of quantal responses in actions tends to induce a strong correlations between the wage outcome and participants' actions. For example, because workers are more likely to accept a job offer at a high wage, the quantal response effect will tend to produce a higher worker expected wage conditional on accepting than the worker expected wage conditional on rejecting an offer. Thus, workers' actions and wages tend to be positively correlated. Similarly, because employers are more likely to offer jobs at lower wages, the quantal response effect will tend to produce a lower employer expected wage conditional on offering than the employer expected wage conditional on not making an offer. Thus, employers' actions and wages tend to be negatively correlated.

In the absence of further constraints maximizing entropy of the joint distribution $f[\omega, A_w, A_c]$

will tend to maximize the differences of expected wages conditional on actions for each agent. In market interactions, however, these correlations are offset by the impact of the action on the outcome. For example, when a worker accepts a job offer that tends to lower the wage for that job, and when a worker rejects an offer it tends to raise the wage for that job. Similarly, when an employer offers a job, that tends to raise the wage for the job and when an employer refrains from making an offer, that tends to lower the wage for that job. To reflect this feedback or impact effect in the constrained maximum entropy framework [Jaynes, 1983], we limit the differences in worker expected wages conditional on accepting and rejecting offers, and the parallel differences in employer expected wages conditional on making and not making an offer, in both cases tending to move the wage relative to an endogenous level, α, which is common to both agents and results from the bargaining process[3]:

$$f_w\left[a_w\right] E\left[\omega-\alpha \left|a_w\right.\right]-f_w\left[\bar{a}_w\right] E\left[\omega-\alpha \left|\bar{a}_w\right.\right] \leq \delta_w \tag{28}$$

$$f_c\left[\bar{a}_c\right] E\left[\omega-\alpha \left|\bar{a}_c\right.\right]-f_c\left[a_c\right] E\left[\omega-\alpha \left|a_c\right.\right] \leq \delta_c \tag{29}$$

Even though worker decisions to reject offers and employer decisions to refrain from making offers are not directly observable in data on wages these constraints are theoretically meaningful and in principle reflect real impacts of decisions on wage levels. The constraint representing the feedback of actions on outcomes is an essential component of the statistical equilibrium that makes the model distinct from the rational inattention framework, which attributes endogenous variation in aggregate outcomes to behavioral constraints alone. Plugging in the quantal response functions we can express the feedback constraints compactly:

$$\int \tanh\left[\frac{\Delta u_w[A_w,\omega]}{2T_w}\right] f[\omega](\omega-\alpha)d\,\omega \leq \delta_w \tag{30}$$

$$\int \tanh\left[\frac{\Delta u_c[A_c,\omega]}{2T_c}\right] f[\omega](\alpha-\omega)d\,\omega \leq \delta_c \tag{31}$$

Because workers and employers interact in the same market the statistical fluctuations of wages experienced in the market will be the same for employers and workers and the feedback constraint can be simplified into a single equation:

[3] This constraint is equivalent to constraining the covariance between the wage and action of each agent if we use the convention $\mathcal{A} = \{1, -1\}$.

$$\int \left(\tanh\left[\frac{\Delta u_w[A_w,\omega]}{2T_w}\right] - \tanh\left[\frac{\Delta u_c[A_c,\omega]}{2T_c}\right]\right) f[\omega](\omega-\alpha)d\,\omega \leq \delta \tag{32}$$

This constraint can also be expressed as the difference of the odds of workers accepting and employers offering weighted by the transaction frequencies because

$$2\left(e^{-\frac{\Delta u_c[A_c,\omega]}{T_c}} - e^{\frac{\Delta u_w[A_w,\omega]}{T_w}}\right)\tau[\omega] = \tanh\left[\frac{\Delta u_w[A_w,\omega]}{2T_w}\right] - \tanh\left[\frac{\Delta u_c[A_c,\omega]}{2T_c}\right] \tag{33}$$

A fixed-point equilibrium is a scalar found by solving a system of equations that is exactly determined by an equal number of unknown variables. A statistical equilibrium solution is a probability distribution that is the solution to a constrained maximization problem. As is well known in physics [Jaynes, 1957] and information theory [Golan, 2018] the least biased statistical equilibrium distribution is the one that maximizes the entropy functional subject to the normalization of probabilities and all relevant constraints. In the labor-market model this implies we maximize the entropy of the joint distribution $f[\omega, A_w, A_c]$ subject to the normalization of probabilities and the feedback constraint:

$$\underset{f[\omega]\geq 0}{\text{Max}} - \int \sum_{\mathcal{A}_w}\sum_{\mathcal{A}_c} f[\omega, A_w, A_c] \log\left[f[\omega, A_w, A_c]\right] d\omega \tag{34}$$

$$\text{subject to} \int \left(\tanh\left[\frac{\Delta u_w[A_w,\omega]}{2T_w}\right] - \tanh\left[\frac{\Delta u_c[A_c,\omega]}{2T_c}\right]\right) f[\omega](\omega-\alpha)d\,\omega \leq \delta \tag{35}$$

$$\text{and} \int \sum_{\mathcal{A}_w}\sum_{\mathcal{A}_c} f[\omega, A_w, A_c]\, d\omega = 1 \tag{36}$$

From the standard identities of the entropy functional we can write the maximum entropy problem in terms of the marginal and conditional distributions and solve for the marginal frequencies of the wage.[4] This formulation provides a useful expression of the model in terms of the marginal frequencies of the observable wage, $f[\omega]$. The joint, conditional, and other marginal distribution can be recovered easily from the laws of probability.

[4]See Appendix A for proofs.

$$\underset{f[\omega]\geq 0}{\text{Max}}\, H = -\int f[\omega]\log[f[\omega]]d\omega + \int f[\omega] H\left[f\left[A_w|\,\omega\right]\right]d\omega + \int f[\omega] H\left[f\left[A_c|\,\omega\right]\right]d\omega \tag{37}$$

$$\text{subject to} \int \left(\tanh\left[\frac{\Delta u_w[A_w,\omega]}{2T_w}\right] - \tanh\left[\frac{\Delta u_c[A_c,\omega]}{2T_c}\right]\right) f[\omega](\omega-\alpha)d\,\omega \leq \delta \tag{38}$$

$$\text{and} \int f[\omega]\,d\omega = 1 \tag{39}$$

The first-order conditions are sufficient as well as necessary to characterize a unique solution because informational entropy, which is the objective function, is strictly concave in frequencies and the constraints are linear in frequency, defining a convex feasible set:

$$f[\omega] = \frac{e^{H[f_w[A_w|\omega]]}e^{-\left(\tanh\left[\frac{\Delta u_w[A_w,\omega]}{2T_w}\right]\frac{(\omega-\alpha)}{S}\right)}e^{H[f_c[A_c|\omega]]}e^{-\left(\tanh\left[\frac{\Delta u_c[A_c,\omega]}{2T_c}\right]\frac{(\omega-\alpha)}{S}\right)}}{\mathbb{Z}[\omega;u_w,u_c,T_w,T_c,S,\alpha]} \tag{40}$$

where $\mathbb{Z}$ is the normalizing constant. Plugging in the payoff functions 6 and 12 the statistical equilibrium wage distribution is proportional to the product of the two agents' QRSE distributions:

$$f[\omega] \propto e^{H[f_w[A_w|\omega]]}e^{-\tanh\left[\frac{\omega-\mu_w}{2T_w}\right]\frac{(\omega-\alpha)}{S}}e^{H[f_c[A_c|\omega]]}e^{-\tanh\left[\frac{\omega-\mu_c}{2T_c}\right]\frac{(\omega-\alpha)}{S}} \tag{41}$$

where $\alpha, \mu_c, \mu_w \in \mathbb{R}$ and $T_c, T_w, S \in (0,\infty)$. In this model μ_c, μ_w are the indifference wages which are the inflection points of between taking one action or another, S represents the market temperature or scale of the feedback, and T_c, T_w represent the behavioral temperature or scale of the behavioral responses and α is the market location or inflection point of the feedback of actions on the outcome. In some settings, such as the statistical equalization of profit rates described in Scharfenaker and Foley [2017] the market location may be determined exogenously by a multitude of factors. In the two-agent market transaction setting, however, the bargaining process between workers and employers is the principle determinate of the central market location for wages. While α will likely tend towards the intersection of the offer curves ω^* described by Eq. 17 as the average of each agents indifference wages weighted by their behavioral temperatures, asymmetric shocks may lead to inertia in the adjustment of α while temporary shocks may lead to no adjustments at all. These cases are examined in the next section.

The statistical equilibrium wage distribution has an important limiting distribution in the zero-entropy case. As all endogenous fluctuations go to zero, defined in terms of the Lagrange multipliers $T_w, T_c, S \to 0$, the wage distribution becomes uniform $\mathcal{U}[\mu_w, \mu_c]$ since transaction probabilities become degenerate and have no feedback on aggregate wages. In this case the unemployment rate is equal to:

$$\lim_{T_w, T_c, S \to 0} u = \frac{p}{p+1} \tag{42}$$

While empirical evidence of non-uniform wage distributions does imply positive entropy in labor-market interactions, it does not imply that it is entirely due to positive decisions temperatures of employers and workers, as in the rational inattention framework. The feedback component that accounts for the impact of labor-market decisions on the aggregate outcomes also defines fluctuations at the market level that lead to non-uniformity of the wage distribution. Because the Lagrange multipliers that determine the model parameters correspond to the market and individual-behavioral components, these effects are in principle identified.

7 Adjustment of Expectations

Equilibrium in expectations is defined by Phelps [1994] as the state in which expectations of market participants are fulfilled. As Bruno de Finetti [de Finetti, 1974] demonstrates, expectations (or previsions) are not predictions, they are probabilistic evaluations of events, and thus cannot be falsified. While employment outcomes themselves cannot falsify an expectation, workers and employers can face numerous other consequences to their actions which can incentivize agents to change their behavior. An important consequence of such behavioral changes in a system comprised of interacting entropy-constrained participants is that the state of the system will also change in response to changes in individual behavior [Foley, 2020b]. When the forces that lead to changes in expectations are absent employers and workers will not revise their expectations absent any unanticipated shocks to the system. Figure 3 depicts the statistical equilibrium wage distribution (left) associated with quantal responses of workers and employers (right). These figures jointly determine the unemployment rate through Eq. 24, which in this case is about 4.5%. This figure depicts equilibrium in expectations, where no endogenous variable will lead to revisions of workers' and employers' expectations.

Now consider an exogenous negative shock to the labor market such as a decline in aggregate demand due to a change in the money supply. Such a shock initially only changes employers' willingness to hire as represented in Figure 4 as a 30% decline in μ_c. In this

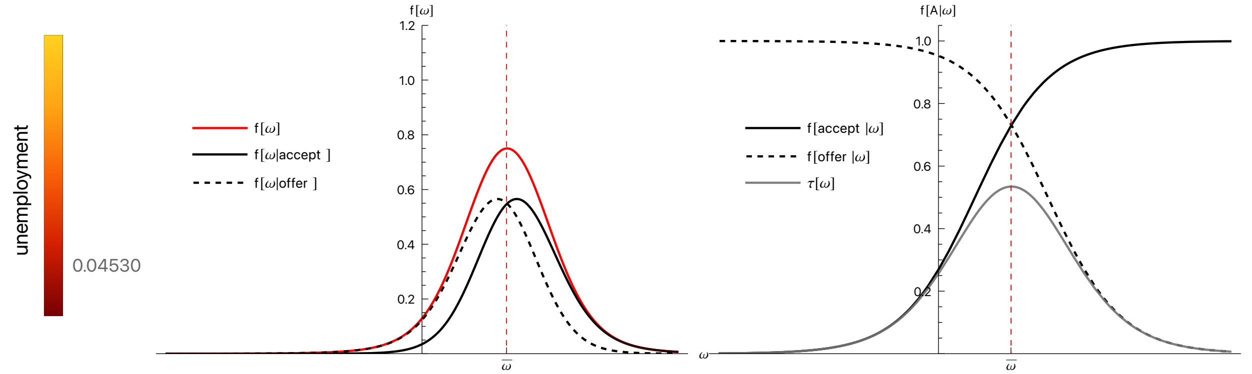

Figure 3: Labor market equilibrium wage distribution and conditional action frequencies for $\mu_w = 0.5, \mu_c = 1.5$, $T_w = 0.5, T_c = 0.5$, $\alpha = 1$m and $S = 1$. In this situation expectations will not be revised and the average market wage $\bar{\omega} = \frac{\mu_c+\mu_w}{2} = \omega^*$.

situation unemployment increases by 1.6% to approximately 6.2%.

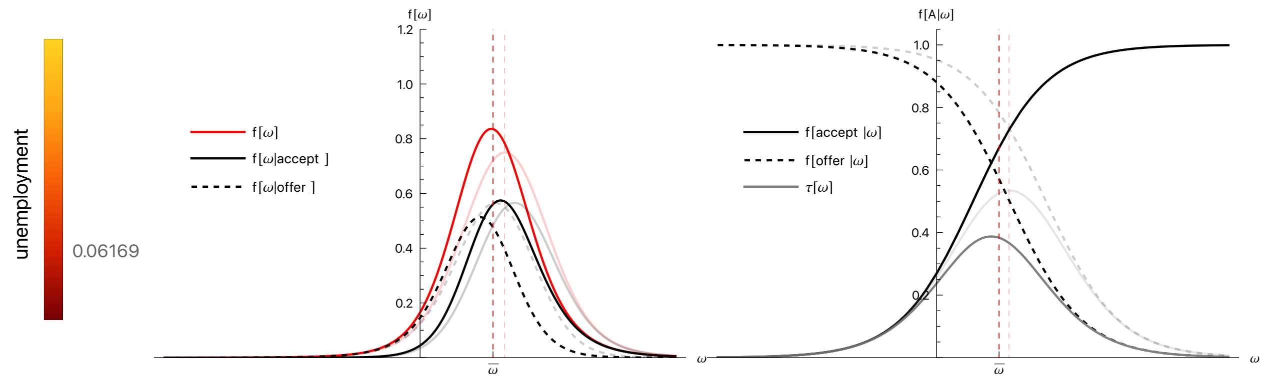

Figure 4: Labor market equilibrium wage distribution and conditional action frequencies after a 30% shock to employers' indifference wage: $\mu_w = 0.5, \mu_c = 1$, $T_w = 0.5, T_c = 0.5$, $\alpha = 1$, and $S = 1$. In this situation the average market wage $\bar{\omega}$ is above the intersection of action frequencies.

If the shock is permanent there will eventually be a proportional adjustment of workers' expectations and a relocation of the average money wage, as captured by a decline in the parameters μ_w and α. Figure 5 demonstrates that at the new equilibrium expectations are once again in equilibrium, albeit at a new lower equilibrium average market wage. Equilibrium unemployment, however, does not return to the pre-shock equilibrium rate of 4.5% as one might expect if money were neutral. Instead, the new equilibrium is defined by both a lower average wage and a higher rate of unemployment.

Unless there is a proportional decline in the behavioral and market scale parameters T_w, T_c and S the new equilibrium unemployment will be higher after agents realign their expectations with the market. Figure 6 shows that only when agents' behavioral response

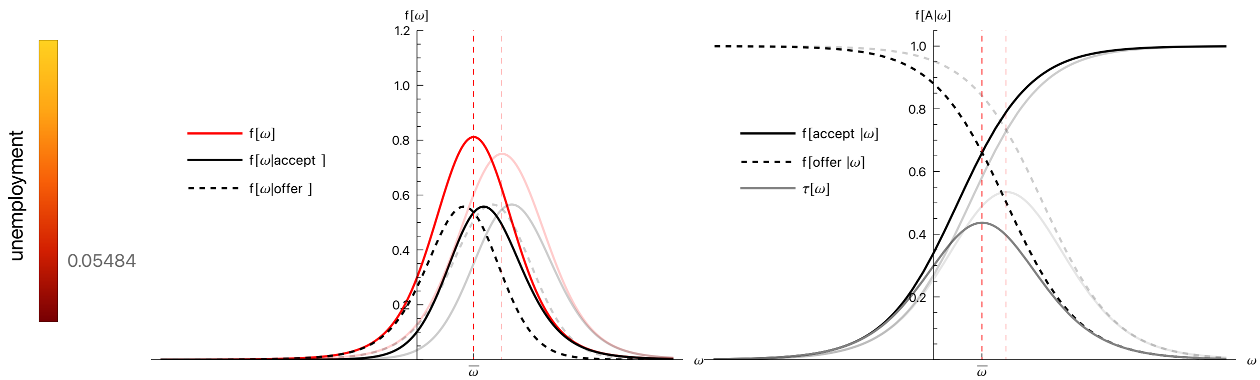

Figure 5: Labor market equilibrium wage distribution and conditional action frequencies after a proportional decline in workers' expectations and market location: $\mu_w = 0.33, \mu_c = 1$, $T_w = 0.5, T_c = 0.5$, $\alpha = \omega^*$, and $S = 1$. In this situation expectations are again fulfilled, but at an average market wage lower than before the shock and at a higher equilibrium rate of unemployment.

temperatures and the market feedback response temperature also decline in proportion to the shock will unemployment and the real wage be neutral to the shock.

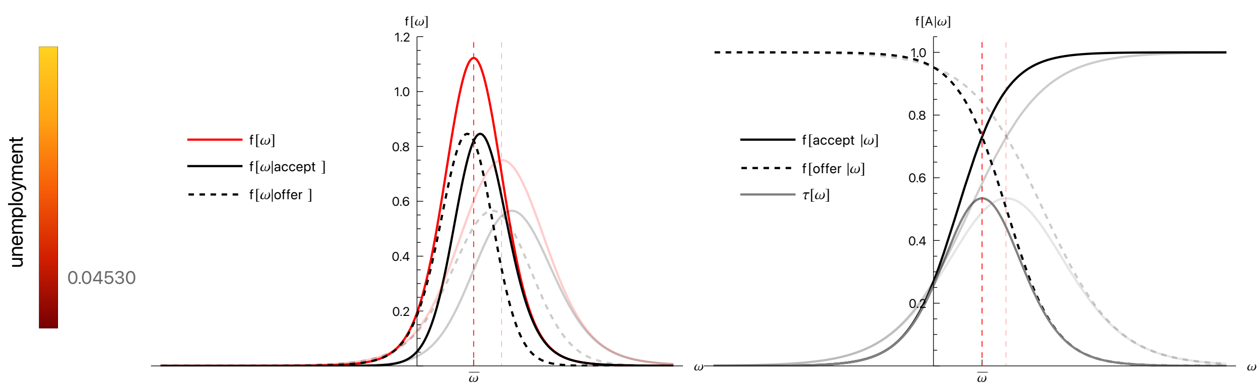

Figure 6: Labor market equilibrium wage distribution and conditional action frequencies after a proportional decline in workers' expectations, market location, and behavioral and market temperatures: $\mu_w = 0.33, \mu_c = 1$, $T_w = 0.33, T_c = 0.33$, $\alpha = \omega^*$, and $S = 0.66$. In this situation expectations are fulfilled and the rate of unemployment and real wage have adjusted to the pre-shock equilibrium rate due to the proportional decline in scale factors T_w, T_c, S.

An important implication of the statistical equilibrium perspective is that the corrections of expectations of the average market outcome by market participants without a proportional correction of response temperatures leads to inertia in the adjustment of the system. In contrast with rational expectations models, which implicitly assume that adjustments are instantaneous and costless and response temperatures are zero, the statistical equilibrium model with informational entropy constrained participants does not predict homogeneity of market outcomes as a consequence of adjustment of expectations of average market outcomes. Because the hypothesis of rational expectations [Friedman, 1968; Muth, 1961] has no analog to the behavioral and market temperatures (which are always implicitly taken to be zero) the equilibrium rate of unemployment and real wage in rational expectations models are invariant to monetary shocks.

8 Discussion

8.1 When is Money Neutral?

The discussion of the neutrality of money as far as we know has up to this point rested on assumptions of the homogeneity of demand and supply curves with respect to the level of prices and wages. This assumption has led economists to the view that a change in a nominal exogenous variable, such aggregate demand responding to a change in the money supply, is analytically equivalent to a change in the denomination of the currency. Under this assumption the neutrality of money in the sense of invariance of real outcomes to changes in the money supply or aggregate demand is guaranteed. Classic analyses of the neutrality of money such as Friedman [1968] invariably invoke this principle.

The current stylized model of the labor market assuming an entropy-constrained form of bounded rationality, however, underlines the importance of another dimension to this question. When currencies are re-denominated, for example, the euro replacing the franc, it is not implausible to suppose that employers and workers (and transactors in other markets) adjust both the level of their expected offers and their expectations of the scale of fluctuations in offers proportionately, which, as we have seen in our model, does lead to the same real outcome in terms of price and wage levels and unemployment. But when aggregate demand changes due to a shock in monetary policy or the broader economic environment, the scale on which employers and workers judge differences in wage offers may not adjust at the same rate as their expectations of the level of wage offers. As we see in the current model, this type of uneven adjustment can lead to real changes in the wage and level of unemployment. Milton Friedman often alluded to "the level of unemployment that would be ground out by the Walrasian system," but neither the Walrasian system nor Friedman's own models of labor market equilibrium addressed the dimension of reactions to fluctuations in wage offers. We can see from the current model that it is precisely this dimension, represented by the "temperature-like" scale factors T_w, T_c, S that "grinds out" equilibrium levels of unemployment. Adjustment of expectations of average wage outcomes can eliminate the involuntary component of unemployment associated with unfulfilled expectations of wage levels, but will not necessarily lead to the same level of frictional unemployment.

8.2 Statistical Equilibrium Unemployment

The statistical equilibrium model of labor market interactions sheds new light on the concepts of involuntary and frictional unemployment. The event space of possible interactions unambiguously defines the voluntary/involuntary attributes of employment/unemployment outcomes, but the statistical component of the model introduces endogenous frictions that can result in voluntary, but non-optimal outcomes.

8.2.1 Involuntary Unemployment

Voluntary and involuntary unemployment are categories that decompose the model's predicted unemployment. The QRSE labor market model defines multiple dimensions of labor market interactions through conditional probabilities. There are four events possible in the encounter between employers and workers described in Table 1.

Voluntary employment is defined as the transaction event in which an employer offers and a worker accepts. These transactions happen according to the conditional probability $f[\text{offer}, \text{accept}|\omega] = f_c[\text{offer}|\omega]f_w[\text{accept}|\omega]$. Voluntary unemployment is defined as the event in which the employer offers and workers turn down employment at a given wage. This event happens according to the conditional probability $f[\text{offer}, \text{turn down}|\omega] = f_c[\text{offer}|\omega]f_w[\text{turn down}|\omega]$.

Involuntary unemployment is defined as the event in which the worker is willing to accept a job at a given wage, but the employer does not offer, which event happens according to the conditional probability $f[\text{not offer}, \text{accept}|\omega] = f_c[\text{not offer}|\omega]f_w[\text{accept}|\omega]$. In the statistical equilibrium model with entropy-constrained behavior, involuntary unemployment is measured probabilistically, since workers/employers have positive probability of accepting/not offering at any given wage. The frequency of *voluntary unemployment* is the event transaction probability times the frequency of workers at a given wage integrated over the wage distribution, which is the area of the joint distribution of employers offering, workers turning down, and the wage:

$$\int_{-\infty}^{\infty} f_c[a_c|\omega]f_w[\bar{a}_w|\omega]f[\omega]d\omega = \int_{-\infty}^{\infty} f[a_c, \bar{a}_w, \omega]d\omega \tag{43}$$

The frequency of *involuntary unemployment* is similarly defined:

$$\int_{-\infty}^{\infty} f_c[\bar{a}_c|\omega] f_w[a_w|\omega] f[\omega] d\omega = \int_{-\infty}^{\infty} f[\bar{a}_c, a_w, \omega] d\omega \tag{44}$$

We can visualize total measurable unemployment decomposed into its voluntary and involuntary components. Figure 7 shows the frequency of voluntary and involuntary unemployment for a given wage and the corresponding behavioral dimension of worker and employee unemployment interactions. In the case of the pre-shock economy above, involuntary unemployment only accounts for about 25% of total measurable unemployment.

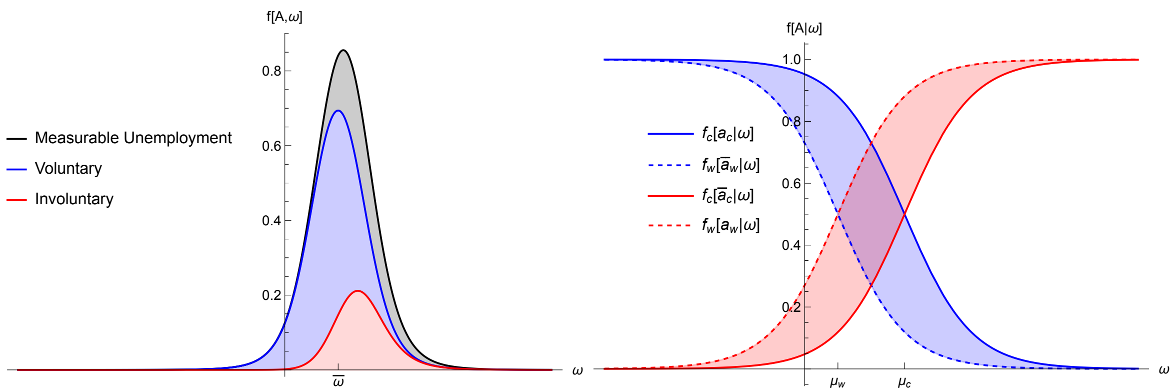

Figure 7: Left: Frequency of total measurable unemployment for a given wage decomposed into voluntary and involuntary unemployment for $\mu_w = 0.5, \mu_c = 1.5$, $T_w = 0.5, T_c = 0.5$, $\alpha = 1$, and $S = 1$. Right: Quantal response curves for workers and employers that define categories of unemployment. With these parameters involuntary unemployment is only about 25% of measurable unemployment.

Involuntary unemployment arises from entropy constrained behavior of workers and employers as well as from the endogenous fluctuations that arise due to the constraint on the feedback of employment interactions on wages. Intuitively, the zero-entropy limit defined by $T_w, T_c, S \to 0$ defines the frictionless equilibrium labor market where the wage distribution is uniform and all unemployment is voluntary.

Following the example above, a negative shock to employers' indifference wage of 30% corresponds to a 1.6% increase of unemployment and an increase of the share of involuntary unemployment from 25% to 32.5%, which is easy to see as employers are much less willing to make offers at wages that workers would likely accept. After the partial adjustment of workers expectations (in terms of a proportional decline in workers' indifference wage) the unemployment rate falls to about 5.5% and involuntary unemployment share declines to about 27%. Only after the proportional adjustment of workers' and employers' expectations of the scale of fluctuations of wages to the actual levels of fluctuations will unemployment and the share of involuntary unemployment return to the pre-shock equilibrium rate. Inertia and uneven adjustment of expectations can lead to permanent changes to the equilibrium rate of unemployment and share of involuntary unemployment.

8.2.2 Frictional Unemployment

Frictional unemployment is the unemployment that arises due to the decentralized interactions of labor market participants and other broader determinants of entropy-constrained behavior, such as incomplete information about the wage distribution, that results in job offers that would have been accepted in a zero-entropy interaction but are turned down. Frictional unemployment arises from positive values of T_w, T_c, and S and can be observed in the difference between the zero- and positive-entropy interactions. As shown in Figure 2, in the limit as $T \to 0$ the quantal response curves reduce to Heaviside step functions which define a zero-entropy rectangle with an area equal to the bid-ask spread $\mu_c - \mu_w$ representing total potential economic surplus. Only in this limit is the indifference wage equivalent to the reservation wage.

With entropy-constrained behavior there are transaction probabilities $\tau[\omega] < 1$ within the zero-entropy rectangle that imply workers are voluntarily turning down offers that exceed their reservation wage. This outcome corresponds to *frictional unemployment*. Similarly there are transaction probabilities $\tau[\omega] > 0$ above and below the reservation wage bounds μ_w and μ_c which would be zero in the frictionless labor market. This outcome implies that workers are accepting jobs below their reservation wage and employers are offering jobs above their reservation wage resulting in *frictional employment*. The area within the zero-entropy rectangle between one and the transaction frequency curve is a counterfactual comparison that defines total frictional unemployment while the area outside of the rectangle and under the transaction frequency curve defines the share of frictional employment.

8.3 Efficiency Wages

The statistical equilibrium model of labor market interactions is also consistent with the existence of efficiency wages because some market transactions will be at wages above workers indifference wages. In general, if there is a mutually advantageous realization of economic surplus through employment transactions there will be a positive average bid-ask spread, which implies the average wage will be above workers' indifference wage. This gap can motivate worker effort through a threat of dismissal. There is an additional perceived cost to job loss that disciplines workers because once unemployed there is a non-zero probability of remaining unemployed. The important difference between the conventional theory of efficiency wages and the statistical equilibrium wage distribution is that in statistical equilibrium the gap between the average wage and indifference wage is not solely the result of an explicit policy of employers. The statistical equilibrium model predicts a positive gap between workers' indifference wage and the average wage independently of employers' intentions. In this sense, the efficiency wage can arise in a statistical sense as an unintended consequence of labor market interactions.

9 Conclusions

Social outcomes can arise from complex market interactions with non-negligible feedbacks that stabilize the state variables into equilibrium frequency distributions. Entropy-constrained behavior that implies endogenous randomness of individual agents' actions in the context of given social and economic institutions casts new light on core problems of macroeconomics, including real wage fluctuations and involuntary unemployment. The quantal response statistical equilibrium distribution is a parsimonious description of the endogenous fluctuations and higher moments of the macroeconomic state variable. Shocks to the system change the parameters of the QRSE distribution that can be easily understood in terms of individual- and system-level behavior. The incorporation of behavioral and market scale parameters help to explain endogenous fluctuations in observable macroeconomic phenomena and the role of expectations and inertia in determining changes in the state of system.

Appendix A: Proofs

Entropy-Constrained Behavior

We assume that workers and employers choose an action from a finite set of actions $A \in \mathcal{A}$ with an associated payoff $u[A, \omega] : \mathcal{A} \to \mathbb{R}$ and maximize their expected payoff subject to a minimum constraint on the entropy of the mixed strategy:

$$\underset{f[A|\omega]\geq 0}{\text{Max}} \sum_{\mathcal{A}} f[A|\omega] u[A, \omega] \tag{A.1}$$

$$\text{subject to} \sum_{\mathcal{A}} f[A|\omega] = 1 \tag{A.2}$$

$$\sum_{\mathcal{A}} -f[A|\omega] \log f[A|\omega] \geq H_{\min}[\omega] \tag{A.3}$$

The Lagrangian associated with this programming problem is:

$$\begin{aligned}\mathcal{L}[f;\lambda,T] = &-\sum_{\mathcal{A}} f[A|\omega]u[A,\omega] - \lambda\left(\sum_{\mathcal{A}} f[A|\omega] - 1\right)\\ &+ T\left(\sum_{\mathcal{A}} f[A|\omega]\log[f[A|\omega]] - H_{min}\right)\end{aligned} \tag{A.4}$$

The first-order conditions require the conditional action frequencies to be distributed according to the Gibbs distribution:

$$f[A|\omega] = \frac{e^{\frac{u[A,\omega]}{T}}}{\sum_{\mathcal{A}} e^{\frac{u[A,\omega]}{T}}} \tag{A.5}$$

This problem has the dual form of maximizing the entropy of the mixed strategy subject to normalization of probabilities and a minimum expected payoff representing "satisficing" bounded rationality:

$$\underset{f[A|\omega]\geq 0}{\text{Max}} \sum_{\mathcal{A}} -f[A|\omega]\log f[A|\omega] \tag{A.6}$$

$$\text{subject to} \sum_{\mathcal{A}} f[A|\omega] = 1 \tag{A.7}$$

$$\sum_{\mathcal{A}} f[A|\omega]u[A,\omega] \geq U_{min}[\omega] \tag{A.8}$$

In this case the first-order conditions require

$$f[A|\omega] = \frac{e^{\beta u[A,\omega]}}{\sum_{\mathcal{A}} e^{\beta u[A,\omega]}} \tag{A.9}$$

The Lagrange multiplier T is the entropy cost of increasing expected payoff, or the terms on which the agent trades off information and expected payoff. The Lagrange multiplier $\beta = 1/T$ is the inverse of the behavior temperature T and has the behavioral interpretation of the expected payoff cost of increasing entropy, or the terms on which the agent trades off expected payoff and information. With two actions $\mathcal{A} = \{a, \bar{a}\}$, the Gibbs distribution reduces to the logistic quantal response function:

$$f[a|\omega] = \frac{e^{\frac{u[a,\omega]}{T}}}{e^{\frac{u[a,\omega]}{T}} + e^{\frac{u[\bar{a},\omega]}{T}}} = \frac{1}{1 + e^{-\frac{u[a,\omega]-u[\bar{a},\omega]}{T}}} = \frac{1}{1 + e^{-\frac{\Delta \mathrm{u}[A,\omega]}{T}}} \tag{A.10}$$

$$f\left[\bar{a}\right|\omega] = 1 - f[a|\omega] = \frac{1}{1 + e^{\frac{\Delta \mathrm{u}[A,\omega]}{T}}} \tag{A.11}$$

Joint, Marginal, and Conditional Entropy

The joint entropy can be written in terms of the marginal and conditional entropies:

$$H\left[\omega, A_w, A_c\right] = H[\omega] + H[A_w|\omega] + H[A_c|\omega] \tag{A.12}$$

where we assume that $H\left[A_w|A_c\right] = H\left[A_w\right]$ so that workers' and employers' decisions are conditionally independent of one another. This assumption only implies that workers and capitalists only interact through the wage ω. The entropy of the actions of workers and employers conditional on the wage is

$$H\left[A_w|\,\omega\right] = \int f[\omega] H\left[f\left[A_w|\,\omega\right]\right] d\omega \tag{A.13}$$

$$H\left[A_c|\,\omega\right] = \int f[\omega] H\left[f\left[A_c|\,\omega\right]\right] d\omega \tag{A.14}$$

The joint entropy can be written as:

$$H\left[\omega, A_w, A_c\right] = -\int f[\omega]\mathrm{Log}[f[\omega]]d\omega + \int f[\omega] H\left[f\left[A_w|\,\omega\right]\right] d\omega + \int f[\omega] H\left[f\left[A_c|\,\omega\right]\right] d\omega \tag{A.15}$$

where $H\left[f_i\left[A_i|\,\omega\right]\right] = -\left(\frac{1}{1+e^{\frac{\Delta u_i[A_i,\omega]}{T_i}}}\mathrm{Log}\left[\frac{1}{1+e^{\frac{\Delta u_i[A_i,\omega]}{T_i}}}\right] + \frac{1}{1+e^{-\frac{\Delta u_i[A_i,\omega]}{T_i}}}\mathrm{Log}\left[\frac{1}{1+e^{-\frac{\Delta u_i[A_i,\omega]}{T_w}}}\right]\right)$ for $i = \{w, c\}$ is the entropy of the conditional action function for workers and employers.

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