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### Abstract

A simple model of activist macroeconomic policy derives a reaction function by assuming that rational governments have performance objectives, but are constrained by the Phillips curve. Although not formally modeled, governments apply a variety of instruments to influence inflation and output, in addition to monetary policy these including fiscal policy, bailouts and exchange rates. Our econometric results are generally consistent with US economic history. One qualification is that governments appear more likely to target growth rates than output gaps. Another inference is that inflation expectations are more likely to be backward than forward looking; a variety of rational expectations models fit the data less well than do simple inertial expectations. We also find that annual data series are more appropriate than quarterly ones for studying these issues.

**JEL codes:** E3, E6

**Keywords:** stabilization policy, inflation targets, expectations

## 1. Introduction

Central to the Keynesian conception of stabilization policy is the assumption that governments actively lean against the macroeconomic wind. This can be thought of as rational behavior for a government constrained by a Phillips curve.<sup>1</sup> A number of alternative specifications are consistent with this theme. One of these relates to the functional form of the government's objective function. Starting with a quadratic form involving an inflation target, we highlight the differences implied by substituting an output growth target for the conventional output gap target.

Another modeling issue concerns how agents and governments make inflation forecasts; we explore several possibilities. We assume that governments are rational throughout, but for agents we begin with simple backward-looking expectations, and develop extensions to forward-looking ones, including rational and new Keynesian expectations. Forward-looking expectations are appealing because they cohere with the notion of well-informed agents. We find, however, that none of our rational expectations specifications improve over the simple inertial model as judged by posterior model probabilities.

A third modeling issue is the timing of policy reactions: how quickly can policy makers respond to nominal and real shocks? A one period is a common lag assumption, but is that lag a quarter or a year? An alternative suggested by Svensson (1997) is a double lag, one for real output and two lags for inflation. We report estimates to distinguish among these possibilities.

## 2. Endogenous stabilization

The monetary policy literature invariably invokes an augmented Phillips curve as a structural constraint on policy makers. Conventionally this is an inverse relation between the unexpected inflation and the gap between actual and natural unemployment. Since the potential output  $Y_t^*$  is conceptually related to

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<sup>1</sup> The original insight for this literature dates to Kalecki (1943). Modern versions begin with Kydland and Prescott (1977) who introduced the logic of rational expectations; Barro and Gordon (1983) further develop this logic.

the equilibrium or natural rate of unemployment, the output gap can be substituted for the unemployment gap as the measure of macroeconomic disequilibrium,

$$\pi_t = E_{t-1}^a \pi_t + \psi x_t + \varepsilon_t, \quad (1)$$

where  $\pi_t$  is the inflation rate,  $x_t \equiv \ln(Y_t) - \ln(Y_t^*)$  is the output gap,  $Y_t$  is real output and  $\varepsilon_t$  an inflation shock. Expected inflation  $E_{t-1}^a \pi_t$  is often interpreted as the forecast of a typical agent based on information available in the previous year. Assuming expectations are fulfilled in the long run, (1) rules out any long-run deviation from  $x = 0$ . However, as long as economic agents do not fully anticipate fiscal, monetary and other policies, governments are able to temporarily increase output at the cost of higher inflation.

Beginning with Fischer (1977) a number of explanations of the Phillips relationship have been offered, including overlapping nominal wage contracts, stochastic price resetting, costly price adjustment and stochastic updating of information. Calvo's (1983) "sticky price" model assumes that only a fraction of all firms are able to adjust its price in the each period. A notable result is that the new Keynesian curve is forward-looking expectations, as contrasted to the backward-looking interpretation typical in textbooks. This paper explores this issue empirically.

Another essential element is an assumption about political objectives. A simple possibility supposes that the government's goals are given as a quadratic function of the output gap and inflation,

$$U = -\left(x^2 + (\pi - \pi^T)^2\right),$$

where  $\pi^T$  is the inflation target, not necessarily the announced target. Social welfare is often defined as some aggregation of individual preferences. Governmental targets may reflect a weighted average of citizen preferences. Woodford (2003) establishes microfoundations for several close relatives of this function form as an approximation to the utility of a representative consumer-worker. Objectives might also

include the discounted value of expected future outcomes.<sup>2</sup> Our approach accounts only for the period in which current policy initially affects outcomes, ignoring other periods as second order.

Quadratic forms are tractable because they result in linear solutions.<sup>3</sup> Within the quadratic family a variety of alternatives are plausible. Ours has circular indifference curves, but these can be made elliptical by adding a parameter to reflect the relative weight of inflation versus output goals. Some studies consider parabolic indifference curves.<sup>4</sup> Differing targets for inflation could account for ideological differences. Often the output target exceeds zero.<sup>5</sup> Kiefer (2008) estimates several different quadratic forms. He finds that it is not possible to statistically separate goal weights from inflation and output targets.<sup>6</sup> Thus, the inflation-target parameter may be interpreted as a composite measure of weights and targets.

Government has limited options in this model. Although it may be able to exploit information advantages to lean against the macroeconomic wind, nevertheless its goals ( $x=0$  and  $\pi=\pi^T$ ) are usually unattainable in the short run. Initially we assume that policymaking is only effective after a one-period delay. Carlin and Soskice (2005) explain this delay with a lag in the *IS* relation between the interest rate and output gap.<sup>7</sup> Recognizing that governments have more tools than just the interest rate, we assume this lag also applies to other instruments. Accordingly, we add an expectations operator and date the objective as

$$E_t^g U = -E_t^g \left( x_{t+1}^2 + (\pi_{t+1} - \pi^T)^2 \right), \quad (2)$$

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<sup>2</sup> The government might plan for its current term of office only, or it might plan to be in office for several terms, discounting the future according to the probability of holding office. Alternatively, it might weigh pre-election years more heavily. These ideas are pursued in Kiefer (2000) who finds little evidence that governments have long-term stabilization goals.

See Svensson (1997) for an elaboration of monetary policy based on multi-period objective functions. See Carlin and Soskice (2005) for a discussion of the single-period simplification.

<sup>3</sup> Ruge-Murcia (2003) questions the conventional linearity assumption. He develops an alternative where the government's inflation preferences are asymmetrical around its target.

<sup>4</sup> See, for example, Alesina et al. (1997).

<sup>5</sup> Barro and Gordon (1983) assume a zero inflation target and an unemployment target below the natural rate.

<sup>6</sup> Also see Ireland (1999).

<sup>7</sup> Although plausible, such policy lags conflict with conventional consumer choice derivations of the *IS* curve which do not show any lag; for example see Galí (2008).

which defines the government's expectation of next period's welfare. Subject to the Phillips curve constraint, the government's preferred inflation is

$$\pi_{t+1}^* = \frac{E_t^a \pi_{t+1} + \psi^2 \pi^T + E_t^s \varepsilon_{t+1}}{1 + \psi^2},$$

To the extent that agents are rational and well informed they would expect this inflation rate (then this solution reduces to  $\pi_{t+1}^* = \pi^T$ ), however if actual forecasts are inertial, then the government has an informational advantage.

Assuming that the government cannot forecast the inflation shock,  $E_t^s \varepsilon_{t+1} = 0$ , lagging one period, and adding two unexpected shocks,  $\varepsilon_t$  and  $\xi_t$ , we obtain the solution for inflation.

$$\begin{aligned} \pi_t &= \frac{E_{t-1}^a \pi_t + \psi^2 \pi^T}{1 + \psi^2} + \varepsilon_t \\ x_t &= \frac{-\psi(E_{t-1}^a \pi_t - \pi^T)}{1 + \psi^2} + \xi_t \end{aligned} \tag{3}$$

The output gap solution follows from substituting the inflation solution back into (1). Initially we take the shocks to be exogenous, independent and unpredictable. Following Carlin and Soskice we take the period of analysis to be one year, although we also investigate a shorter period of one quarter.

Rational agents come to understand that a policy of  $\pi^T = 0$  implies inflation. In the absence of shocks, the time-consistent equilibrium inflation rate should occur where inflation is just high enough so that the government is not tempted to spring a policy surprise. This equilibrium occurs at zero output gap and the inflation target,  $x = 0$  and  $\pi = \pi^T$ . Equations (3) are reduced forms determined by  $E_{t-1}^a \pi_t$ ,  $\xi_t$  and  $\varepsilon_t$ ; they are linear in the variables, but nonlinear in coefficients.

We assume that the government implements policy through variety of instruments (monetary policy, unemployment insurance, tax rebates, infrastructure spending, bailouts, etc.) and that the various agencies pursue this common policy. We assume that policy can be parameterized as a fixed inflation target. Our model can be seen to be the first two equations of Carlin and Soskice's three-equation model, ignoring the *IS* equation. We would need several equations to directly model the government's instruments;

we would need to assume that these all can be separated from the underlying reaction functions as the *IS* curve can, and initially that all display the same one period lag structure.

In comparison to the literature on monetary policy econometrics this is a small and stylized specification. Recent research reports much more complicated dynamic stochastic general equilibrium models (DSGE); see Christiano et al. (2005) or Smets and Wouters (2003). For example, Smets and Wouters specify 4 structural parameters without estimation and estimate 32 additional parameters in a 9-equation model. Their approach includes habit formation in consumption, technology and preference shocks, capital adjustment costs and less than full capacity utilization; it also accounts for sticky prices and wages, along with markups deriving from market power. Our 2-equation model follows the DSGE approach by applying Bayesian estimation methods, but it estimates only 2 parameters.<sup>8</sup> We propose that a traditional Phillips curve can approximate the more complicated equilibrium resulting from sticky prices and from technology and preference shocks. Although the DSGE literature includes detailed descriptions of consumer and firm objectives and behavior, it often models government behavior without an objective function as an agnostic stochastic process, or as a Taylor rule.<sup>9</sup> Our approach focuses on government goals. Although we do not elaborate microfoundations for the rest of the economy, our 2-equation model is dynamic and does have an equilibrium.

### 3. Growth targets

We also consider a related objective function specified on growth rates, rather than output gaps. Thus, we rewrite output in terms of the growth rate as

$$E_t^s U = -E_t^s \left( (g_{t+1} - g_{t+1}^*)^2 + (\pi_{t+1} - \pi^T)^2 \right).$$

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<sup>8</sup> For a discussion of the appropriateness of this estimation method, see Fernandez-Villaverde and Rubio-Ramirez (2004).

<sup>9</sup> Although Adolfson et al. (2011) specify a quadratic objective function more general than ours, they discard it in favor of an *ad hoc* Taylor-type rule for interest rates.

Although this specification is uncommon in the literature, it is appropriate if voters more concerned about the growth rate than the level of output. Woodford (2003) derives a similar form from microfoundations under the assumption that the representative citizen's utility exhibits habit persistence.

The output gap and the growth rate are related concepts; the growth rate can be expressed as

$$g_t \equiv \ln(Y_t) - \ln(Y_{t-1}) \equiv g_t^* - x_{t-1} + x_t,$$

where  $g_t^* \equiv \ln(Y_t^*) - \ln(Y_{t-1}^*)$  is the growth of potential output. Substituting this identity into to the growth target objective function, we derive government policy as before

$$\begin{aligned} \pi_t &= \frac{E_{t-1}^a \pi_t + \psi^2 \pi^T + \psi x_{t-1}}{1 + \psi^2} + \varepsilon_t \\ x_t &= \frac{x_{t-1} - \psi(E_{t-1}^a \pi_t - \pi^T)}{1 + \psi^2} + \xi_t \end{aligned} \tag{4}$$

Comparing the solutions (3) versus (4), we see that the growth-target solution adds an inherited condition, the lagged value of the output gap.

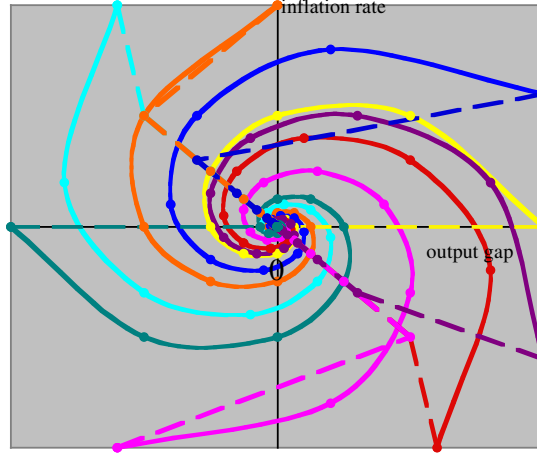
#### 4. Expectations

Many economists view backward-looking expectations with suspicion because they lack microfoundations, and because their forecasts can be irrational. Nevertheless we begin with the inertial approximation for expected inflation,  $E_{t-1}^a \pi_t = \pi_{t-1}$ . This simple forecasting rule, common in textbooks, has the desirable property that it converges to the time-consistent equilibrium. Under this assumption dramatic dynamic differences can arise from the output and growth-target objective functions, as illustrated in Figure 2. The conventional output-target dynamic jumps to a single southeast-tending trajectory in response to a positive inflation shock (northwest-tending with negative inflation shocks), regardless of the output shock. This southeast-northwest path (along with the inclusion of an *IS* curve relating interest rates to the output gap) facilitates the derivation of Taylor-type rules. With a growth target, policy making is more complicated.



Figure 2. Generic convergence trajectories: output target dashed, growth target solid

$$E_{t-1}^a \pi_t = \pi_{t-1}, \psi = 1, \pi^T = 0$$



The typical agent might know the government’s inflation target and objective function; a rational agent would use this information to forecast inflation. Taking the expectation of (3), we can show that the model-consistent expectation is  $E_{t-1}^a \pi_t = \pi^T$ . Alternatively, if the typical agent knows that government pursues a growth target, we take the expectation of (4) finding that  $E_{t-1}^a \pi_t = \pi^T + \frac{x_{t-1}}{\psi}$ .

#### 5. Data, posterior means and model probabilities

Figure 1 plots the data that we wish to explain, OECD (2011) estimates of the US GDP gap and inflation rate measured as both quarterly and annual series. Although quarterly series are usually studied in the macroeconomic literature, this plot suggests that the annual series may convey a nearly equivalent amount of information.

Figure 1. US data GDP deflator inflation (pink) and the GDP gap (blue), annual (thick) and quarterly (thin) observations, 1965-2010

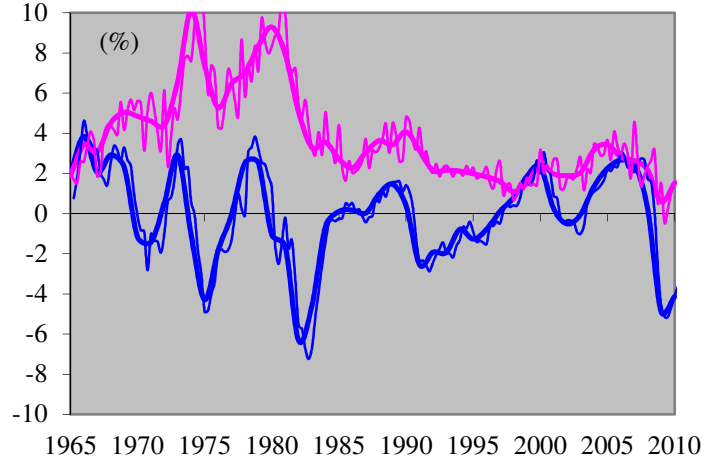


Table 1 reports our initial results. Model (a) estimates our basic output-target model according to (3) assuming inertial expectations,  $E_{t-1}^a \pi_t = \pi_{t-1}$ . Model (b) is the growth-target model (4), also assuming inertial expectations. (c) assumes model-consistent rational expectations and an output target; we stipulate that these forecasts are made in the previous year. Similarly, (d) assumes model-consistent rational expectations for a growth target. Our estimation procedure uses a first order Taylor expansion around each model's steady state solution to approximate the contemporary solution (which may depend on expectations about the future). This solution is obtained using only on the past state of the system and current shocks. Table 1 reports posterior means estimated with the Metropolis-Hastings algorithm.

The first column lists of our assumptions about the prior distributions of the parameters. For the slope and target we take the prior distributions to be normal. Estimates of the slope of the Phillips curve in the literature vary widely, see Schorfheide (2008). We take that the prior mean of the slope to be 0.5.<sup>10</sup> We take the prior mean of the inflation target to be 2%, a number frequently mentioned in press. Perhaps this is unrealistically low for the early years of our data set, an issue that we explore further below. The standard deviations of the shocks have inverted gamma distributions.

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<sup>10</sup> For our annual data the OLS estimate of the augmented Phillips curve (1) under the inertial expectation assumption gives a slope of 0.30 with a standard deviation of 0.06.

When we attempt to estimate these models with less dogmatic priors we sometimes fail to achieve convergence for some of the less likely specifications, like models (a) and (c). With (c) the typical agent uses her knowledge of government's inflation target to rationally forecast inflation so that the economy converges immediately to its equilibrium. It can be shown that under model-consistent expectations (3) reduces to

$$\begin{aligned}\pi_t &= \pi^T + \varepsilon_t \\ x_t &= \xi_t\end{aligned}$$

In this case the slope is not identified, although the target still is. Thus, it is not surprising that the posterior distribution of the slope for model (c) is nearly identical to its prior.

Table 1. Initial estimation results, standard deviations in parentheses, 45 annual observations, 1965-2010

|  | prior            | (a)                            | (b)                               | (c)                            | (d)                               |
|--|------------------|--------------------------------|-----------------------------------|--------------------------------|-----------------------------------|
|  |                  | output target,<br>inertial exp | growth<br>target,<br>inertial exp | output target,<br>rational exp | growth<br>target,<br>rational exp |
|  | mean<br>(SD)     | mean<br>(SD)                   | mean<br>(SD)                      | mean<br>(SD)                   | mean<br>(SD)                      |
| Phillips curve slope $\psi$                      | 0.500<br>(2.000) | 0.343<br>(0.116)               | 0.326<br>(0.080)                  | 0.477<br>(1.991)               | 3.557<br>(0.995)                  |
| inflation target $\pi^T$                         | 2.000<br>(3.000) | 3.206<br>(0.983)               | 3.168<br>(0.876)                  | 3.783<br>(0.333)               | 3.802<br>(0.336)                  |
| price shock std. deviation, $\sigma_\varepsilon$ | (1.000)          | (1.133)                        | (1.086)                           | (2.254)                        | (2.245)                           |
| output shock std. deviation, $\sigma_\xi$        | (1.000)          | (2.190)                        | (1.821)                           | (2.279)                        | (2.281)                           |
| posterior model probability                      |                  | 0.00046                        | 0.99954                           | 0.00000                        | 0.00000                           |

In all cases the estimated slopes of the Phillips curve are positive, but not always statistically significant. The estimated target implies equilibrium inflation rates between 3 and 4 percent. Based on posterior model probabilities, the data clearly support the growth target over the output target, and prefer inertial inflation expectations assumption to the rational assumption.

## 6. New Keynesian Phillips curve

A different version of rational expectations follows Calvo's (1983) sticky-price model of the Phillips curve, a stochastic derivation renown for its microfoundations. This new Keynesian Phillips curve model assumes that firms are uncertain whether they can reset their price; some firms receive a random

“price-change signal” each period. Since resetting firms may be unable to reset again for some time, they rationally forecast future conditions weighing future periods according to the price-resetting probability. The essential conclusion of this analysis is to rewrite (1) in terms of forward-looking expectations ( $E_t^a \pi_{t+1}$  rather than  $E_{t-1}^a \pi_t$ ). New Keynesian expectations are decided in the current period, without the one period lag that we assume for models (c) and (d). Our test of this theory simply substitutes this forward-looking expectation into (3) and (4).

### 7. Double-lag timing

Some authors assume that the government can implement policy remedies for inflation and output shocks without any lag.<sup>11</sup> Another possibility stipulates a double-lag: the output impact is delayed by one period as before, but the inflation impact is delayed by two periods. Svensson (1997) hypothesizes that output is affected by policy after one period, and inflation effects are delayed an additional period due to the lagging of output gap in the Phillips curve,

$$\pi_t = E_{t-1}^a \pi_t + \psi x_{t-1} + \varepsilon_t, \quad (5)$$

although he offers no theoretic argument for these lags.

For this alternative we date the arguments according to when they can be impacted by current policy,<sup>12</sup>

$$E_t^g U = -E_t^g \left( x_{t+1}^2 + (\pi_{t+2} - \pi^T)^2 \right)$$

Solving by the same method, lagging appropriately and adding random shocks to both the inflation and output solution gives

$$\begin{aligned} \pi_t &= \frac{E_{t-2}^g (E_{t-1}^a \pi_t) + \psi^2 \pi^T}{1 + \psi^2} + \varepsilon_t \\ x_t &= \frac{-\psi (E_{t-1}^g (E_t^a \pi_{t+1}) - \pi^T)}{1 + \psi^2} + \xi_t \end{aligned} \quad (6)$$

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<sup>11</sup> For example, Clarida et al. (1999) specify an *IS* curve in which current interest rates determine current outputs.

<sup>12</sup> For simplicity we do not discount the inflation term even though that it would be appropriate for this dating.

where  $E_{t-2}^g(E_{t-1}^a \pi_t)$  denotes the government's expectation in  $(t-2)^{th}$  period of the private sector's forecast to be formed in the  $(t-1)^{th}$  period. This double-lag timing assumption implies that inflation is affected by the government's two-period forecast of inflation. A two-period government forecast also affects output, but here it is only one period old. Policy is forward looking in this model, taking into account future inflation.

The growth-target function also can be used to derive double-lag versions. Under the double-lag timing assumption, a growth-target objective results in

$$\begin{aligned}\pi_t &= \frac{E_{t-2}^g(E_{t-1}^a \pi_t) + \psi^2 \pi^T + \psi x_{t-2}}{1 + \psi^2} + \varepsilon_t \\ x_t &= \frac{x_{t-1} - \psi(E_{t-1}^g(E_t^a \pi_{t+1}) - \pi^T)}{1 + \psi^2} + \xi_t\end{aligned}\tag{7}$$

Again the solution depends on two-period forecasts.

Table 2 compares five alternative expectations models. Models (e) and (f) estimate the new Keynesian Phillips curve, substituting  $E_t^a \pi_{t+1}$  for  $E_{t-1}^a \pi_t$  in (3) for an output target and in (4) for a growth target. Consistent with our previous approach, our double-lagged specification assumes that government expectations are rational. Although an alternative double-lagged model might define agent expectations as inertial, Table 2 report estimates (g) and (h) for which both governments and agents are assumed to have model-consistent rational expectations, according to (6) for an output target, and (7) for a growth target. As a basis for comparison, we also repeat our best-fitting specification (b) from Table 1, the growth-target model under inertial expectations. Our results show that none of these alternatives perform better than (b). Carlin and Soskice (2005) show that the double-lag timing (assuming an output target and adding an *IS* equation) facilitates the derivation a Taylor rule. The poor result for model (g) casts doubt on this theoretical approach to Taylor rules.

Table 2. Alternative expectation models, standard deviations in parentheses, 45 annual observations, 1965-2010

|  | prior            | (b)                         | (e)                   | (f)                   | (g)                     | (h)                     |
|--|------------------|-----------------------------|-----------------------|-----------------------|-------------------------|-------------------------|
|  |                  | growth target, inertial exp | output target, NK exp | growth target, NK exp | output target, two lags | growth target, two lags |
|  | mean (SD)        | mean (SD)                   | mean (SD)             | mean (SD)             | mean (SD)               | mean (SD)               |
| Phillips curve slope $\psi$                      | 0.500<br>(2.000) | 0.326<br>(0.080)            | 3.557<br>(0.995)      | 2.272<br>(0.864)      | 0.477<br>(1.991)        | 2.541<br>(2.859)        |
| inflation target $\pi^T$                         | 2.000<br>(3.000) | 3.168<br>(0.876)            | 3.802<br>(0.336)      | 3.807<br>(0.349)      | 3.783<br>(0.333)        | 3.773<br>(0.342)        |
| price shock std. deviation, $\sigma_\varepsilon$ | (1.000)          | (1.086)                     | (2.245)               | (2.334)               | (2.254)                 | (2.168)                 |
| output shock std. deviation, $\sigma_\xi$        | (1.000)          | (1.821)                     | (2.281)               | (2.420)               | (2.279)                 | (2.295)                 |
| posterior model probability                      |                  | 0.99999                     | 0.00000               | 0.00000               | 0.00001                 | 0.00000                 |

## 6. Serial correlation

There is a methodological problem with the above results: all models assume uncorrelated error terms, despite the frequent observation of serial correlation in macroeconomic time series.<sup>13</sup> Thus, we modify the error terms to introduce first-order autocorrelation. With serial correlation rational policymakers can forecast the inflation shock and should modify their intervention strategy. Solving by the same method for an output gap target gives

$$\begin{aligned}\pi_t &= \frac{E_{t-1}^a \pi_t + \psi^2 \pi^T + E_{t-1}^s \varepsilon_t}{1 + \psi^2} + \varepsilon_t \text{ where } \varepsilon_t = \rho_\varepsilon \varepsilon_{t-1} + \mu_t \\ x_t &= \frac{-\psi(E_{t-1}^a \pi_t - \pi^T - E_{t-1}^s \varepsilon_t / \psi^2)}{1 + \psi^2} + \xi_t \text{ where } \xi_t = \rho_\xi \xi_{t-1} + v_t\end{aligned}\quad (8)$$

Notice that the government's expectation of the inflation shock now enters both equations. A growth target now gives

$$\begin{aligned}\pi_t &= \frac{E_{t-1}^a \pi_t + \psi^2 \pi^T + E_{t-1}^s \varepsilon_t + \psi x_{t-1}}{1 + \psi^2} + \varepsilon_t \text{ where } \varepsilon_t = \rho_\varepsilon \varepsilon_{t-1} + \mu_t \\ x_t &= \frac{x_{t-1} - \psi(E_{t-1}^a \pi_t - \pi^T - E_{t-1}^s \varepsilon_t / \psi^2)}{1 + \psi^2} + \xi_t \text{ where } \xi_t = \rho_\xi \xi_{t-1} + v_t\end{aligned}\quad (9)$$

<sup>13</sup> Clovis and Zhang (2010) demonstrate the importance of accounting for autocorrelation in the error structure of the Phillips curve. The DSGE literature customarily introduces serial correlation into the shocks affecting pricing equations; typical estimates report quite high autocorrelation parameters.

With double-lag timing assumption and serial correlation for an output-gap targets gives

$$\begin{aligned}\pi_t &= \frac{E_{t-2}^g(E_{t-1}^a \pi_t) + \psi^2 \pi^T + E_{t-2}^g \varepsilon_t}{1 + \psi^2} + \varepsilon_t \text{ where } \varepsilon_t = \rho_\varepsilon \varepsilon_{t-1} + \mu_t \\ x_t &= \frac{-\psi(E_{t-1}^g(E_t^a \pi_{t+1}) - \pi^T - E_{t-1}^g \varepsilon_{t+1} / \psi^2)}{1 + \psi^2} + \xi_t \text{ where } \xi_t = \rho_\xi \xi_{t-1} + \mu_t\end{aligned}\tag{10}$$

and for a growth target

$$\begin{aligned}\pi_t &= \frac{E_{t-2}^g(E_{t-1}^a \pi_t) + \psi^2 \pi^T + E_{t-2}^g \varepsilon_t + \psi \alpha_{t-2}}{1 + \psi^2} + \varepsilon_t \text{ where } \varepsilon_t = \rho_\varepsilon \varepsilon_{t-1} + \mu_t \\ x_t &= \frac{x_{t-1} - \psi(E_{t-1}^g(E_t^a \pi_{t+1}) - \pi^T - E_{t-1}^g \varepsilon_{t+1} / \psi^2)}{1 + \psi^2} + \xi_t \text{ where } \xi_t = \rho_\xi \xi_{t-1} + \mu_t\end{aligned}\tag{11}$$

Table 3 repeats the specifications of Table 1 with this alternative error structure. Our prior assumption is that both autocorrelation parameters follow the uniform distribution  $U(0,1)$ .<sup>14</sup> Repeating the best-fitting uncorrelated specification (b) for comparison, we find strong evidence of autocorrelation implying that multiple shocks are often strung together. Again, the growth-target model (j) has the highest posterior probability, again inertial expectations performs best. We find that allowing for serial correlation increases our estimate of the slope dramatically. We noted above the wide range of slope estimates in the literature, but (j)'s estimate is an order of magnitude greater than those in the DSGE literature. Even without a good explanation for this result, the good fit of (j) calls into question the rather flat slope estimates obtained by other researchers.

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<sup>14</sup> We find that we need to tighten our prior on the inflation target in order to achieve estimation convergence for some of the less likely specifications in Tables 3 and 4.

Table 3. Modifying the initial models for serial correlation, standard deviations in parentheses, 45 annual observations, 1965-2010

|  | prior            | (b)                         | (i)                               | (j)                               | (k)                               | (l)                               |
|--|------------------|-----------------------------|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|
|  |                  | growth target, inertial exp | output target, inertial exp, auto | growth target, inertial exp, auto | output target, rational exp, auto | growth target, rational exp, auto |
|  | mean (SD)        | mean (SD)                   | mean (SD)                         | mean (SD)                         | mean (SD)                         | mean (SD)                         |
| Phillips curve slope $\psi$                      | 0.500<br>(2.000) | 0.326<br>(0.080)            | 1.306<br>(0.652)                  | 2.664<br>(0.671)                  | 1.925<br>(0.807)                  | 3.074<br>(1.113)                  |
| inflation target $\pi^T$                         | 2.000<br>(2.000) | 3.168<br>(0.876)            | 3.378<br>(0.825)                  | 3.215<br>(0.963)                  | 3.304<br>(0.932)                  | 3.086<br>(1.110)                  |
| inflation autocorrelation $\rho_\varepsilon$     | 0.500<br>(0.289) |                             | 0.566<br>(0.218)                  | 0.834<br>(0.084)                  | 0.768<br>(0.104)                  | 0.873<br>(0.066)                  |
| output autocorrelation $\rho_\xi$                | 0.500<br>(0.289) |                             | 0.651<br>(0.127)                  | 0.605<br>(0.132)                  | 0.657<br>(0.122)                  | 0.599<br>(0.148)                  |
| price shock std. deviation, $\sigma_\varepsilon$ | (1.000)          | (1.086)                     | (1.085)                           | (1.013)                           | (1.016)                           | (1.013)                           |
| output shock std. deviation, $\sigma_\xi$        | (1.000)          | (1.821)                     | (1.748)                           | (1.688)                           | (1.800)                           | (1.850)                           |
| posterior model probability                      |                  | 0.00034                     | 0.00457                           | 0.99385                           | 0.00124                           | 0.00000                           |

Table 4 extends the models of Table 2 in a similar fashion. Here we repeated the best-fitting model (j) from Table 3 as a basis of comparison. We find that (j) has the highest posterior probability, and note that the new Keynesian, growth target (n) has the second-best fit to these data. None of the other rational expectations models are supported by these data, in fact, some result in unexpected negative slope estimates. Our results raise doubts about the DSGE literature that customarily discards inertial expectations as not worthy of consideration.

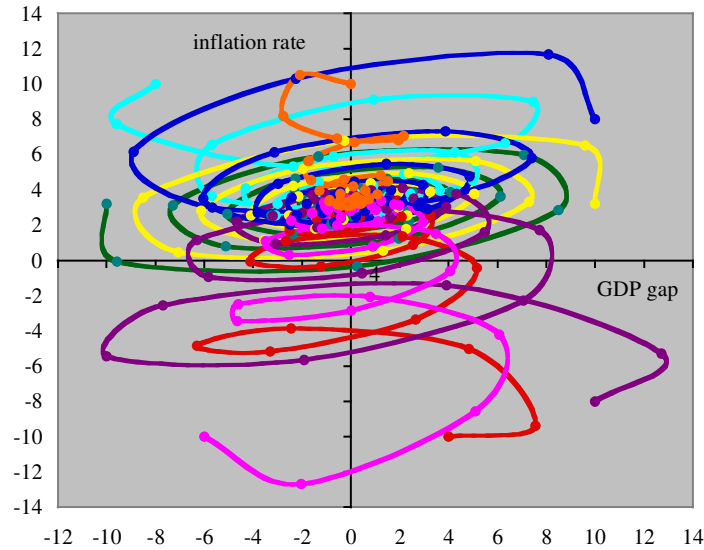


Table 4. Modifying the alternative expectation models for serial correlation, standard deviations in parentheses, 45 annual observations, 1965-2010

|  | prior            | (j)                               | (m)                         | (n)                         | (o)                           | (p)                           |
|--|------------------|-----------------------------------|-----------------------------|-----------------------------|-------------------------------|-------------------------------|
|  |                  | growth target, inertial exp, auto | output target, NK exp, auto | growth target, NK exp, auto | output target, two lags, auto | growth target, two lags, auto |
|  | mean (SD)        | mean (SD)                         | mean (SD)                   | mean (SD)                   | mean (SD)                     | mean (SD)                     |
| Phillips curve slope $\psi$                      | 0.500<br>(2.000) | 2.664<br>(0.671)                  | -0.996<br>(0.594)           | 2.150<br>(0.845)            | -0.858<br>(0.766)             | -1.349<br>(1.856)             |
| inflation target $\pi^T$                         | 2.000<br>(2.000) | 3.215<br>(0.963)                  | 3.362<br>(0.945)            | 3.182<br>(1.018)            | 3.271<br>(1.056)              | 3.293<br>(1.341)              |
| inflation autocorrelation $\rho_\varepsilon$     | 0.500<br>(0.289) | 0.834<br>(0.084)                  | 0.819<br>(0.076)            | 0.878<br>(0.059)            | 0.363<br>(0.243)              | 0.425<br>(0.282)              |
| output autocorrelation $\rho_\xi$                | 0.500<br>(0.289) | 0.605<br>(0.132)                  | 0.670<br>(0.116)            | 0.494<br>(0.150)            | 0.679<br>(0.114)              | 0.318<br>(0.247)              |
| price shock std. deviation, $\sigma_\varepsilon$ | (1.000)          | (0.963)                           | (0.550)                     | (0.788)                     | (1.134)                       | (1.210)                       |
| output shock std. deviation, $\sigma_\xi$        | (1.000)          | (1.691)                           | (1.723)                     | (2.103)                     | (1.809)                       | (1.891)                       |
| posterior model probability                      |                  | 0.92970                           | 0.00002                     | 0.07028                     | 0.00001                       | 0.00000                       |

The dynamics of the US economy are considerably more complicated than those depicted in textbooks. Figure 3 plots the responses of our best-fitting model to a variety of initial conditions, comparable to the trajectories plotted in Figure 1. This analysis illustrates the rich dynamics that can arise from our simple approach. These response paths do show convergence, although they suggest that the economy can worsen before it improves, and that it follows counterclockwise looping paths toward its eventual equilibrium.

Figure 3. Estimated trajectories: model (j), growth target, inertial expectation, and serial correlation



#### 8. Quarterly data

To further explore timing issues, we re-estimate these models with quarterly observations. The literature rarely addresses the impact of different temporal aggregations and customarily studies quarterly data.<sup>15</sup> Redefining the period as a quarter shortens reaction time dramatically and leads to dramatically different estimation results. Table 5 reports only the posterior means of the parameter estimates and the posterior probabilities. The data overwhelmingly favor model (b), the growth-target model with inertial expectations and without autocorrelation. (b)'s estimated slope is nearly flat, while our annual results find slopes that are much steeper. Substituting these estimates into the model equations implies that inflation and the output gaps are essentially independent random walks. Probably this result (random walks do not converge to equilibrium) explains why we find them so difficult to estimate. Five of the serial correlation specifications fail to achieve convergence. A flat slope is consistent with those reported in the DSGE literature, see Schorfheide (2008), although this literature does not compare its estimates to random walks.

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<sup>15</sup> For an exception, see Paya et al. (2007) who compare inflation persistence for the monthly, quarterly and annual cases. With the exception of Ruge-Murcia (monthly) and Kiefer (annual) all of the other empirical studies cited in this paper use quarterly data.

One interpretation of our results might be that governments do not attempt stabilization at all, but another is that the conventional choice of quarterly data for macroeconomic policy analysis fails to account for the yearlong lags inherent in the policy making. We favor the latter.

Table 5. Abbreviated summary for all specifications using 185 quarterly observations, 1965Q2-2011Q2<sup>16</sup>

| model                                 | Phillips curve slope $\psi$ | inflation target $\pi^T$ | inflation auto. $\rho_\varepsilon$ | output auto. $\rho_\xi$ | posterior probability |
|---------------------------------------|-----------------------------|--------------------------|------------------------------------|-------------------------|-----------------------|
| (a) output target, inertial exp.      | 0.262                       | 3.270                    |                                    |                         | 0.000                 |
| (b) growth target, inertial exp.      | 0.027                       | 1.873                    |                                    |                         | 1.000                 |
| (c) output target, rational exp.      | 0.414                       | 3.039                    |                                    |                         | 0.000                 |
| (d) growth target, rational exp.      | 6.475                       | 3.831                    |                                    |                         | 0.000                 |
| (e) output target, NK exp.            | 0.496                       | 3.769                    |                                    |                         | 0.000                 |
| (f) growth target, NK exp.            | 0.039                       | 3.855                    |                                    |                         | 0.000                 |
| (g) output target, two lags           | 0.982                       | 3.767                    |                                    |                         | 0.000                 |
| (h) growth target, two lags           | 3.397                       | 3.792                    |                                    |                         | 0.000                 |
| (i) output target, inertial exp, auto | nonconvergent               |                          |                                    |                         |                       |
| (j) growth target, inertial exp, auto | 5.820                       | 3.431                    | 0.889                              | 0.936                   | 0.000                 |
| (k) output target, rational exp, auto | nonconvergent               |                          |                                    |                         |                       |
| (l) growth target, rational exp, auto | 0.939                       | 2.144                    | 0.957                              | 0.838                   | 0.000                 |
| (m) output target, NK exp, auto       | nonconvergent               |                          |                                    |                         |                       |
| (n) growth target, NK exp, auto       | 5.605                       | 3.380                    | 0.900                              | 0.935                   | 0.000                 |
| (o) output target, two lags, auto     | nonconvergent               |                          |                                    |                         |                       |
| (p) growth target, two lags, auto     | nonconvergent               |                          |                                    |                         |                       |

### 9. Evolution of the inflation target

Throughout we have assumed that the inflation target is a constant parameter. This ignores the literature on structural shifts in stabilization doctrine.<sup>17</sup> An evolving-target generalization can be defined letting  $\pi_t^T$  be a state variable in a state space model. We redefine the target in the best-fitting (j) specification as a random walk  $\pi_t^T = \pi_{t-1}^T + \omega_t$ , and we estimate its parameters by conventional methods.<sup>18</sup> This generalization is motivated by the conjecture that the inflation target has evolved over time, while

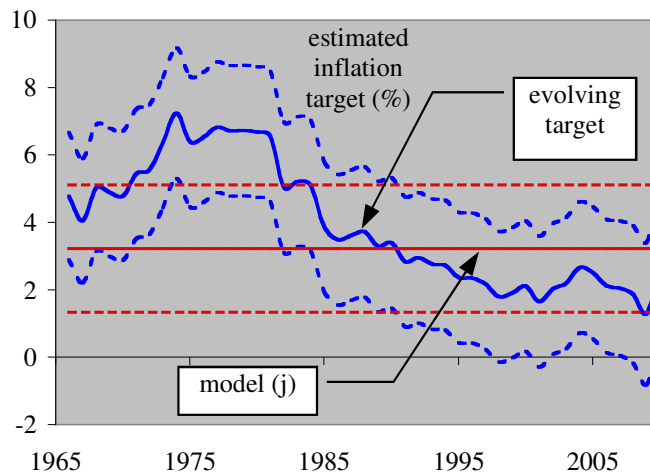
<sup>16</sup> All models impose somewhat tighter priors in order to achieve convergence; even so we do not achieve convergence for five models. In all cases we assume normal distributions for the slope of the Phillips curve, 0.5 (1) and for the inflation target, 2 (2) and uniform distributions for the inflation autocorrelation parameter (0.5,1) and the output autocorrelation parameter (0,1). And again we assume unit standard errors for the shock terms and inverse gamma distributions.

<sup>17</sup> See for example Clarida et al. (2000) or Estrella and Fuhrer (2003).

<sup>18</sup> Our random walk model of the target introduces a unit root into the model, which is not permitted by the software we use to estimate the models reported above.

remaining agnostic about its path. We assume that  $\omega_t \sim N(0, \sigma_\omega^2)$ , and let  $\sigma_\omega^2$  be a parameter to be estimated. This method requires an assumption of a prior for the target; we stipulate  $\pi_{10}^T = 3\%$  with a variance of 9. With this specification the maximum likelihood estimate for the slope is 1.99 (0.39), somewhat less than that of (j), but still steep. Figure 4 plots our smoothed estimate of the evolving target. Its path is less than smooth because our estimate of the standard deviation of the random target step is rather high at 0.82. Our estimate peaks at above 7% in 1974 and declines to almost 1% near the end of the sample period. Although the wide 95% confidence interval suggests considerable uncertainty about this parameter, Figure 4 seems plausible in light of the evolution of stabilization doctrine. We hesitate to draw conclusions about the evolution government's target because this evolving-target specification fits the data less well than the fixed-target version (j), which estimates a constant target of about 3%.<sup>19</sup>

Figure 4. Estimating the government's inflation target as a random walk, dashed lines for the 95% confidence intervals



## 10. Conclusion

We begin with a Keynesian characterization of stabilization policy, a simple model involving an augmented Phillips curve and a simplified theory of government behavior. Overall, the Keynesian notion of

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<sup>19</sup> The Schwartz criterion is 7.36 for the evolving-target model, and 7.33 for model (j) re-estimated by maximum likelihood.

activist governments who lean against the macroeconomics wind is consistent with the US evidence; using annual data we estimate an inflation target of around 3 percent with a rather steep Phillips curve. One contribution is our finding that governments have been more likely to target growth rates, than output gaps. A second finding is that rational and forward-looking inflation expectations do not improve the fit of the model as compared to backward-looking ones. Finally, our results are consistent with the inference that the application of stabilization policy effects inflation and output together after a one year lag, and that the conventional use quarterly data to study these issues overlooks the longer lags inherent in stabilization policy.

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