There are 100 points possible on this exam, 50 points each for Prof. Lozada's questions and Prof. Govindan's questions.

There are three sections on this exam:

- In the first section contains all of the required questions. There are four of them. The first two (from Prof. Lozada) are worth 18 points each; the last two (from Prof. Govindan) are worth 15 points each.
- In the second section there are two questions from Prof. Govindan; you should work one of them. Each is worth 20 points.
- In the third section there are two questions from Prof. Lozada; you should work one of them. Each is worth 14 points.

You have 4 hours and 30 minutes (that is, until 1:30 PM) to finish this test. This gives you about 45 minutes per question.

Do not use different colors in your answers because we may grade looking at black-and-white photocopies of your exam.

Answers with illegible or difficult-to-read-handwriting may lower your grade because we may not be able to read and understand your answers, especially considering that we are looking at copies. So it is in your best interest to make your answers LEGIBLE.

Please put the number of the problem you are working on at the top of every page of your answers, so we do not accidentally ignore part of your answer.

In this document some questions begin on one page and end on the next page; therefore, do not assume that a question ends at the bottom of a page, but check to determine whether it continues onto the next page.

If you think there is some ambiguity in a question, spell out exactly how you interpret that question.

A correct answer will not receive full credit without supporting explanations. Show all your work to receive full credit.

Good luck.

1. Answer all of the following four questions.

1. **[18 points]** Suppose a consumer gets utility from a consumption good c and leisure z ("z" to prevent notation confusion with other words in this problem beginning with the letter 'l,' and because "leisure" has a "z" sound in the middle). Suppose the consumer's utility function is $u(c, z) = cz$.

   The consumer has a maximum amount of time available per day, denoted by $\overline{L}$, divided between leisure and work. Suppose the wage rate is w and the price of c is p.

   (a) (3 points) Find the utility-maximizing ("Marshallian") demand for consumption and for leisure, following this procedure:
   - use a budget constraint rewritten so that $\overline{L}$ appears all by itself on one side of the equation; and
   - introduce a new variable, "α," defined by $\alpha = p/w$, and use it to eliminate p and w from the problem. For the rest of this entire problem (not just part (a) but all the parts), do not use p or w in equations, use α.

   Call the "Marshallian" demand for consumption "c^M" and call the Marshallian demand for leisure "z^M." Do not check second-order conditions.

   (b) (3 points) Find the expenditure-minimizing ("Hicksian") demand for consumption and for leisure. Call these c^H and z^H (the "H" stands for "Hicksian"). Use the correct formula for "expenditure" in this problem, which is $\alpha c + z$, and let the constant in the constraint be $\bar{u}$.

   (c) (1 point) From your answers to part (b), find the minimized level of expenditure as a function of α and $\bar{u}$. Call this level of expenditure e^*. (In this problem, "e" will always mean "expenditure," not the base of the natural logarithms.)

   (d) (1 point) It turns out that e^* should be equal to $\overline{L}$. Why?

   (e) (2 points) There is an identity, written with some omissions as

   $$z^H(\alpha, \bar{u}) = z^M(\text{some stuff}), \tag{1}$$

   which when differentiated yields

   $$\frac{\partial z^H}{\partial \alpha} = \frac{\partial z^M}{\partial \alpha} + \frac{\partial z^M}{\partial e}\frac{\partial e}{\partial \alpha}. \tag{2}$$

   By filling in the "some stuff" in (1), state what that identity is.

(f) (3 points) Equation (2) can be rewritten as

$$\frac{\partial z^M}{\partial \alpha} = \frac{\partial z^H}{\partial \alpha} - \frac{\partial z^M}{\partial e}\frac{\partial e}{\partial \alpha}\,. \tag{3}$$

Using your answers from earlier parts of this problem such as (a) and (b), show that (3)'s left-hand side is equal to its right-hand side. In evaluating the last term on the right-hand side, start by using the fact given in part (d) to eliminate $\overline{L}$.

(g) (3 points) Without using the algebraic expression you found above for c^H, prove that (3) can be rewritten as

$$\frac{\partial z^M}{\partial \alpha} = \frac{\partial z^H}{\partial \alpha} - c^H \cdot \frac{\partial z^M}{\partial e}\,. \tag{4}$$

(h) (1 point) Identify which term of (4) is the income effect, and describe in which direction the income effect would move leisure demand z^M if the wage went up.

(i) (1 point) Identify which term of (4) is the substitution effect, and describe in which direction the substitution effect would move leisure demand z^M if the wage went up.

2. **[18 points]** Consider a pure exchange economy with two persons named "a" and "b" and two commodities named "1" and "2." The utility function and endowment of person "a" are given by

$$u_a = \ln(x_{1a}x_{2a}) \qquad \boldsymbol{\omega_a} = (1, 0)$$

and the utility function and endowment of person "b" are given by

$$u_b = \ln(x_{1b}x_{2b}) \qquad \boldsymbol{\omega_b} = (0, 2).$$

(a) (9 points) Find the competitive general equilibrium prices and quantities in this economy.

(b) (6 points) Find the Pareto Optimal allocations in this economy.

(c) (1 point) Is the competitive general equilibrium in this economy Pareto Optimal? Should it be?

(d) (2 points) What is the core of this economy? (Sketching an Edgeworth Box might help in answering this.)

3. **[15 points]** A person has an expected utility function of the form $u(w) = \sqrt{w}$. He initially has wealth of \$4. He has a lottery ticket that will be worth \$12 with probability 1/2 and will be worth \$0 with probability 1/2.

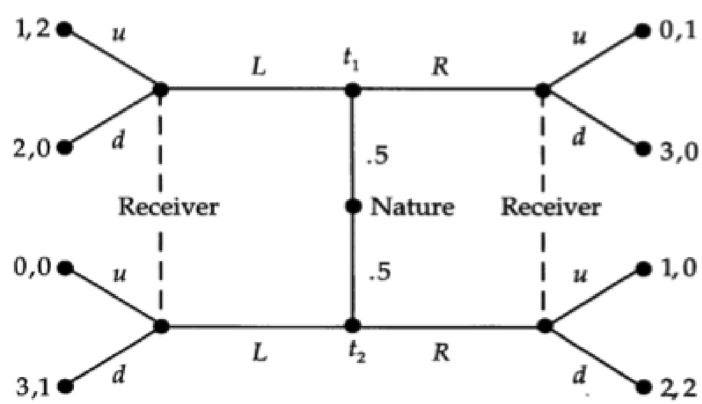
   (a) (2.5 points) What is his expected utility?

   (b) (5 points) What is the lowest price p_l at which he would part with the ticket?

   (c) (5 points) Now suppose he does not initially own the ticket, and has to consider whether to buy one. What is the highest price p_h that he would be willing to pay to buy the ticket?

   (d) (2.5 points) Are buying and selling prices equal? Give an economic interpretation of your answer.

4. **[15 points]** Specify a pooling perfect Bayesian equilibrium in which both Sender types play R in the following signaling game.



2. Answer one of the following two questions.

1. **[20 points]** Consider the following asymmetric information model of the Bertrand duopoly with differentiated products. Demand for firm i is $q_i(p_i, p_j) = a - p_i - b_i \cdot p_j$. The costs are zero for both firms. The sensitivity of firm i's demand to firm j's price is either high or low. That is, b_i is either b_H or b_L, where $b_H > b_L > 0$. For each firm, $b_i = b_H$ with probability θ and $b_i = b_L$ with probability $1 - \theta$, independent of the realization of b_j. Each firm knows its own b_i but not its competitor's. All of this is common knowledge.

   (a) (8 points) What are the strategy spaces, type spaces, beliefs, and utility functions in this game?

   (b) (12 points) What conditions define a symmetric pure-strategy Bayesian Nash equilibrium of this game? Solve for such an equilibrium. *You will get full points for setting up and doing the first-order condition(s) correctly for the firms. No need to solve the algebra.*

2. **[20 points]** Three oligopolists operate in a market with inverse demand given by $P(Q) = a - Q$, where $Q = q_1 + q_2 + q_3$ and q_i is the quantity produced by firm i. Each firm has a zero marginal cost of production and no fixed cost. The firms choose their quantities as follows: (1) firm 1 chooses $q_1 \geq 0$; (2) firms 2 and 3 observe q_1 and then simultaneously choose q_2 and q_3, respectively. What is the subgame-perfect outcome?

3. Answer one of the following two questions.

1. **[14 points]** Suppose a firm uses inputs x_1 and x_2—which we shall collectively call $\mathbf{x}$—to produce output according to a production function $f(\mathbf{x})$. Further suppose:

   - the firm takes the price of output as a constant, p;
   - the firm takes the price of x_1 as a constant, w; and
   - the firm takes the price of x_2, called "q," as a function of the amount of input 2 which it buys: $q(x_2)$. Suppose the more of x_2 the firm wishes to purchase, the higher the price it must pay to buy x_2.

   Finally, suppose the firm's second-order conditions for profit maximization are fulfilled.

   (a) (8 points) Find the expression for how the firm's purchases of x_1 vary when x_1's price varies.

   (b) (3 points) Under what situations is this firm's input demand curve for input 1 downward-sloping?

   (c) (3 points) How does your answer to part (b) change if the supply curve which the firm faces for input 2, namely $q(x_2)$, is linear?

2. **[14 points]** A firm produces output y from inputs x_1 and x_2 whose prices are w_1 and w_2, respectively. The firm takes these input prices as given. The production function is $y = x_1^{1/2} x_2^{1/2}$.

   (a) (2 points) This firm has what kinds of returns to scale?

   (b) (7 points) Find this firm's cost function and state any relevant second-order conditions for doing so. You do not have to confirm that those second-order conditions hold.

   (c) (5 points) Determine whether "the algebraic function you found in part (b)" (namely the cost function) is concave in $\mathbf{w}$.