Summer 2010 Microeconomics Qualifying Exam

There are 72 points possible on this exam, 36 points each for Prof. Lozada's questions and Prof. Kiefer's questions. However, Prof. Lozada's questions are almost equally weighted (they are worth 11 points, 13 points, and 12 points), while Prof. Kiefer's required question is worth 18 points, which is twice as much as his optional questions.

There are three sections on this exam:

- In the first section there are three questions; you should work all of them. The first is worth 11 points; the second is worth 13 points; and the last one is worth 18 points.
- In the second section there are two questions; you should work one of them. Each is worth 12 points.
- In the third section there are three questions; you should work two of them. Each is worth 9 points.

You have 4 hours and 30 minutes (that is, until 1:30 PM) to finish this test. This gives you about 45 minutes per question.

Good luck.

Section 1.
Answer all of the following three questions.

1. [**11 points**] Tell me everything you know about:

   (a) quasiconcavity; and

   (b) why it is important in consumer theory (you do not have to say anything about producer theory).

   The essay you write should be understandable to an undergraduate student who has had no mathematics beyond the first year of calculus; so you will certainly have to define any terms you use which I taught this year.

   You do not have to discuss quasiconvexity, nor dual functions (so there are two reasons why you do not have to discuss the result that "the indirect utility function $v(\mathbf{p}, m)$ is quasiconvex in $\mathbf{p}$").

2. [**13 points**, divided as: 3 points for (a), (e), and (f) taken together; 3 points for (b) and (c) taken together; 3 points for (d); 4 points for (g).]
   In an exam from a previous year, I (Prof. Lozada) asked the following question:

   > Consider a two-person, two-commodity economy in which "x_{ij}" represents the amount of commodity i belonging to person j. Suppose the utility function of person 1 is
   >
   > $$\ln x_{11} + \ln x_{21}$$
   >
   > and the utility function of person 2 is
   >
   > $$\ln x_{12} + \ln x_{22}\,.$$
   >
   > Suppose the initial endowments of persons 1 and 2 are $\omega_1 = (1, 1)$ and $\omega_2 = (2, 1)$, respectively. Find the core of this economy.

   I have attached the two-page handwritten answer to this question right after page 9 of your exam. Please look at those two pages.

   (a) After the old answer sheets are two pages with computer-drawn graphs. You should ignore all of the typewritten computer commands on this page, which are written in a language I do not

expect you to be familiar with. Find Fig. 1. Its horizontal axis is, as labeled, $U^1 = \ln(x_{11} \cdot \frac{2}{3}x_{11})$, and its vertical axis is, as labeled, $U^2 = \ln[(3-x_{11}) \cdot (2-\frac{2}{3}x_{11})]$. As indicated, the graph shows what happens in this (U^1, U^2) space as x_{11} goes from $\sqrt{6}/2$ to $3-\sqrt{3}$, in other words, as x_{11} goes approximately from 1.23 to 1.27. Give an economic interpretation of this curve in (U^1, U^2) space. Hint 1: I have never shown you a graph in (U^1, U^2) space before, so I am not asking you to remember something, I am asking you to interpret something which is new to you. Hint 2: Look back to the old answer sheet, especially the bottom of its second page, for help.

(b) All the remaining parts below pertain to finding the core of the "2-replica" of the above economy, in other words, finding the core of the economy with two people who are identical to Person 1 above and two people who are identical to Person 2 above.
To begin, explain why one of the first steps in finding the core of the 2-replica might be to solve

$$\max[\alpha \ln(x_{11}\, x_{21}) + (1-\alpha) \ln(x_{12}\, x_{22})]$$

for $\alpha \in [0,1]$ such that

$$2x_{11} + x_{12} = 4 \quad \text{and}$$
$$2x_{21} + x_{22} = 3\,.$$

Read part (c) before you answer part (b).

(c) Next, explain why one of the first steps in finding the core of the 2-replica might be to solve

$$\max[\alpha \ln(x_{11}\, x_{21}) + (1-\alpha) \ln(x_{12}\, x_{22})]$$

for $\alpha \in [0,1]$ such that

$$x_{11} + 2x_{12} = 5 \quad \text{and}$$
$$x_{21} + 2x_{22} = 3\,.$$

(d) Solve the problem in part (b). Hint: one way of expressing the answer is

$$x_{11} = 2\alpha$$
$$x_{12} = 4 - 4\alpha$$
$$x_{21} = \tfrac{3}{2}\alpha$$
$$x_{22} = 3 - 3\alpha\,.$$

Next, argue that not all $\alpha \in [0,1]$ are economically relevant, and that the economically relevant α's are

$$\alpha \in \left[\frac{1}{\sqrt{3}}, 1-\frac{1}{\sqrt{6}}\right] \approx [0.59, 1.41]\,.$$

Hint: When I worked the last sentence of this part, one of my intermediate steps was to solve $6\alpha^2 - 12\alpha + 5 > 0$. In solving that it was helpful to remember that if $ax^2 + bx + c = 0$ then $x = (-b \pm \sqrt{b^2 - 4ac})/2a$.

(e) Find Fig. 2 on the first sheet of computer-drawn graphs. It is related to parts (b) and (d) above. Its horizontal axis is, as labeled, $U^1 = \ln(2\alpha \cdot \frac{3}{2}\alpha)$, and its vertical axis is, as labeled, $U^2 = \ln\big[(4-4\alpha)\cdot(3-3\alpha)\big]$. As indicated, the graph shows what happens in this (U^1, U^2) space as α goes from $\frac{1}{\sqrt{3}}$ to $1-\frac{1}{\sqrt{6}}$. Give an economic interpretation of this curve in (U^1, U^2) space.

(f) Find Fig. 3 on the first sheet of computer-drawn graphs. It is related to part (c) above. Guess what Fig. 3 shows. Hint: I give part of the answer away in the first two sentences of part (g), so you might want to read them before answering.

(g) Find Fig. 4, on the second sheet of computer-drawn graphs. Fig. 4 just superimposes Figs. 1, 2, and 3. I have indicated what values of x_{11} correspond to two important points in Fig. 4.

   i. Explain the economic conclusions which come from Fig. 4. Then:

   ii. draw an Edgeworth Box, as on the second page of the old exam answer;

   iii. in this Edgeworth Box, show the core of the 2-replica of the economy; and

   iv. in this Edgeworth Box, show how the core of the 2-replica compares to the core of the original economy; and finally,

   v. explain why this result (the result of subpart (iv) which you just completed) is just what one would expect.

3. [18 points]

Airbus produces jet planes y by combining capital x_1 and labor x_2.

(a) Airbus's cost function is

$$c = y\sqrt{w_1 w_2},$$

where w_1 is the rental rate for capital and w_2 is the wage rate. Explain the difference between the cost function above and the one below:

$$c = w_1 x_1 + w_2 x_2 .$$

(b) What is Airbus's production function? Is it homogeneous of degree 1? Monotonic? Convex? Quasiconcave? Sketch the isocost and isoquant curves; give an interpretation the slopes for this case.

(c) Suppose that the rental rate for capital is $w_1 = 2$; the wage rate is $w_2 = 2$, and that demand is given by the function

$$p(y) = 8 - y.$$

And, suppose that this market is competitive. What is its equilibrium? What combination of capital and labor would Airbus use?

(d) Now suppose two firms (Airbus and Boeing) share this market; they behave as a Cournot duopoly. They have identical costs. What is the equilibrium?

(e) Airbus and Boeing still share the market, but now form a cartel. Suppose that they split the cartel quantity and profits equally. And assume that the cartel and Cournot quantities are the only possibilities. Fill in the payoff matrix.

profit payoffs: (Boeing, Airbus)		Airbus	
		cartel	Cournot
Boeing	cartel		
	Cournot		

(f) Suppose that this game is repeated infinitely many times. Consider the *mutual punishment* threat:

- play cartel in the first game; thereafter play cartel,
- unless the either rival plays Cournot in any previous game, then play Cournot.

Under what circumstances is the (cartel, cartel) outcome a subgame-perfect Nash equilibrium of the infinitely repeated game?

(g) Finally, suppose that these two firms behave as a Stackelberg duopoly: only one game is played, Airbus chooses his quantity first, and Boeing follows. What is Stackelberg equilibrium?

(h) Construct a welfare analysis of the cartel, Cournot and Stackelberg equilibriums to determine which is the more efficient market structure. Illustrate your answer. What are the implications of this example for public policy?

Section 2.
Answer one of the following two questions.

1. [12 points] Suppose a firm produces one output. The output is produced using one purchased input called "x," but production is adversely affected by air pollution, the amount of which is called "d" (for "$d$irty air"). Let the price of x be "w."

   (a) How is this firm's purchases of x changed by a change in the amount of air pollution? Find a symbolic answer to this question, then speculate about its sign.

   (b) How is this firm's profits affected by a change in the amount of air pollution?

2. [12 points] Consider a profit-maximizing firm that produces a good which is sold in a competitive market. It is observed that when the price of the output good rises, the firm hires more skilled workers but fewer unskilled workers. Now the unskilled workers unionize and succeed in getting their wages increased. Assume that all other prices remain constant.

   (a) What will happen to the firm's demand for unskilled workers?

   (b) What will happen to the firm's supply of output?

Section 3.
Answer two of the following three questions.

1. [9 points]

(a)

> The postulate of *methodological individualism* underlies all public choice analysis. In trying to explain governmental actions, we begin by analyzing the behavior of the individuals who make up the government. In a democracy these are the voters, their elected representatives, and appointed bureaucrats. The postulate of methodological individualism has a normative analogue. The actions of government *ought* to correspond, in some fundamental way, to the preferences of the individuals who these actions effect, the citizens of the state. The postulate of *normative individualism* underlies much of normative analysis in public choice.
>
> —Dennis Mueller

Explain the term methodological individualism. Discuss its role in neoclassical microeconomics. Give specific examples.

(b)

> No one pretends that democracy is perfect.... Indeed, it has been said that democracy is the worst form of government except all those other forms that have been tried from time to time.
>
> —Winston Churchill

Discuss the limitations of normative individualism.

(c) Discuss and evaluate the alternative schools of thought concerning methodology.

2. [9 points]
Imagine a 2 by 2 economy. There are two consumers, $i = 1$ (Robinson) and 2 (Friday); each consumes two goods, leisure x_{1i} and burritos x_{2i}. Their preferences are identical,

$$u(x_{1i}, x_{2i}) = x_{1i} - \frac{(2x_{2i} - 3)^2}{2}.$$

Their endowments $\omega_i = (\omega_{1i}, \omega_{2i}) = (2, 0)$ are also identical.

Burritos can be produced according to the production function $y_2 = |y_1|$. General equilibrium is described by $x_{1i} = \omega_{1i} + y_{1i}$, $y_{11} + y_{12} = y_1$ and $x_{21} + x_{22} = y_2$. Define the price of leisure as the numeraire, $p_1 = 1$.

(a) Suppose that Robinson and Friday share burrito profits equally; each has a 50% share in the firm. In perfect competition what is equilibrium price of a burrito and the allocation $(x_{11}, x_{21}, x_{12}, x_{22})$? What are profits?

(b) Consider a pure monopoly regime for the burrito market, and assume that Robinson receives all profit. Robinson owns the burrito firm. What is equilibrium price of a burrito and the allocation? Now what are profits?

(c) Consider a reform of the Robinson-monopoly regime in favor of perfect competition. Is this a Pareto improvement?

3. [9 points]
Barbie and Ken consume a private good, coffee x_i, and a public good, poetry G. The utility functions and endowments are of the fixed-proportions form,

$$\begin{array}{lll} \text{Barbie} & U_b = \min(x_b, G) & \omega_b = 3, \\ \text{Ken} & U_k = \min(x_k, G) & \omega_k = 3. \end{array}$$

Endowments are also shown. Each citizen may make a contribution g_i toward the provision of poetry, but such contributions reduce private consumption according to the budget constraint

$$\omega_i = x_i + g_i \,.$$

The private good can be transformed into the public one according to the transformation function

$$x_b + x_k + G - \omega_b - \omega_k = 0 \,.$$

Finally, Barbie and Ken agree on the Benthamite social welfare function,

$$W = U_k + U_b \,.$$

(a) Plot reaction curves in g_k — g_b space. Show that the Nash equilibrium occurs at $(G, x_b, x_k) = (2, 2, 2)$.

(b) Add indifference curves to your g_k — g_b diagram. Given these endowments, find the Pareto set.

(c) Show that the Nash Equilibrium is also the Benthamite social optimum, given these endowments. Illustrate your answer in U_k — U_b space.